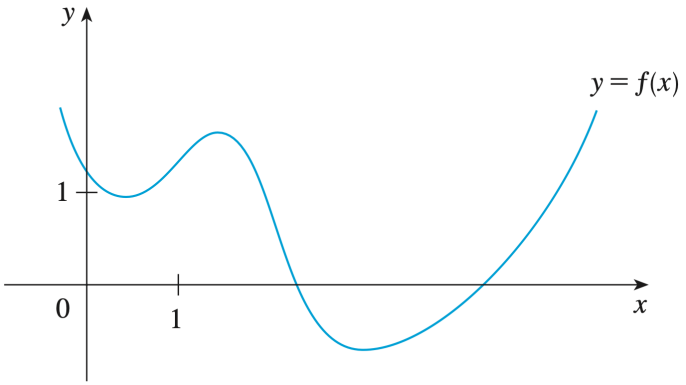
2.8 The Derivative as a Function

Definition. The derivative of a function f at a number x is defined by

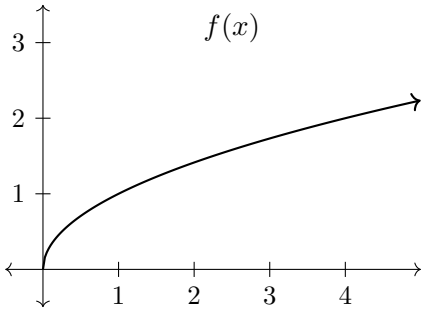
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

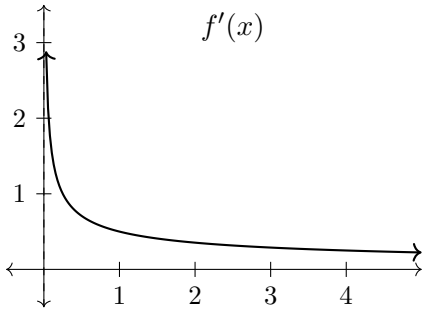
provided this limit exists. The value of f' at x, namely $f'(x)$, can be interpreted geometrically as the slope of the tangent line to the graph of f at the point $(x, f(x))$.

Example. Below is the graph of a function $f(x)$. Sketch the graph of the derivative $f'(x)$.



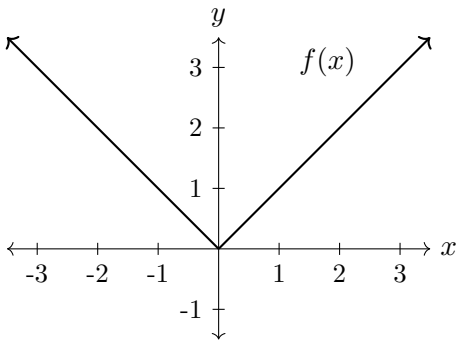
Example. If $f(x) = \sqrt{x}$, find the derivative of f. What is the domain of f'?

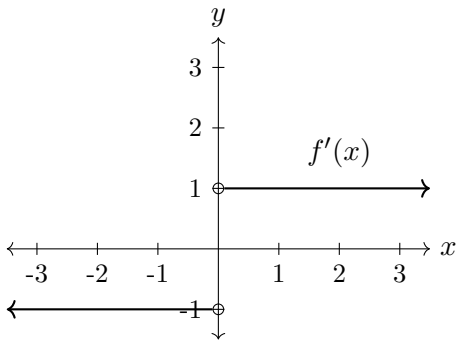




Definition. A function f is **differentiable at** a if $f'(a)$ exists. It is **differentiable on an open interval** (a, b) $\big[$or (a, ∞), or $(-\infty, a)$, or $(-\infty, \infty)\big]$ if it is differentiable at every number in the interval.

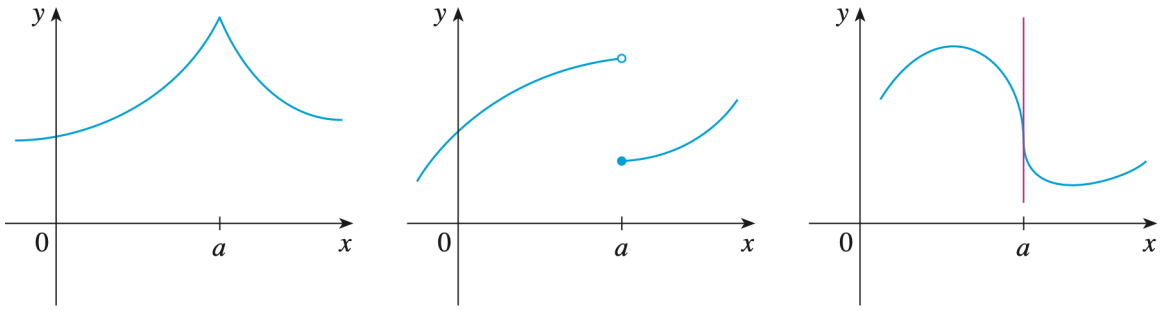
Example. Where is the function $f(x) = |x|$ differentiable?





Theorem. What can we say if f is differentiable at a?

Question. How can a function fail to be differentiable?



(a) A corner (b) A discontinuity (c) A vertical tangent