2.1 The tangent and velocity problems

Example. Galileo found that the distance fallen by any freely falling body (neglecting air resistance) after t seconds is $s(t) = 4.9t^2$ meters. Suppose that a ball is dropped from 450m above the ground. Our goal is to find the velocity of the ball after *exactly* 5 seconds.

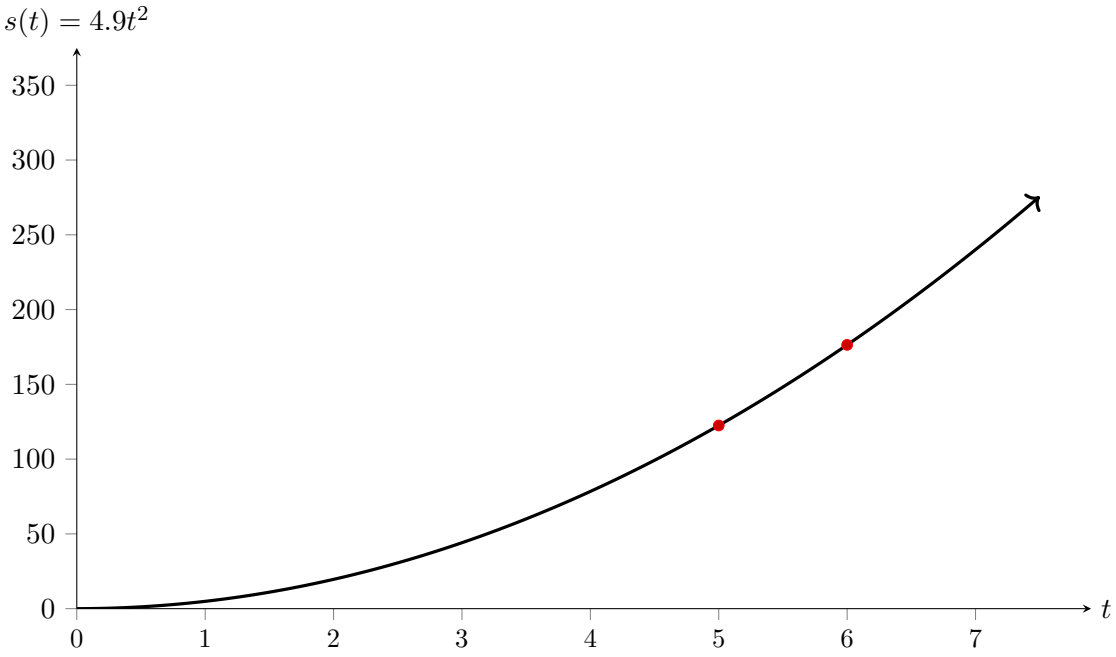
Question. How do you compute the *average velocity* over a time interval $[a, b]$?

Question. Find the average velocity of the ball over the given time intervals.

Time Interval	Average Velocity
$5 \leq t \leq 6$	
$5 \leq t \leq 5.1$	
$5 \leq t \leq 5.05$	
$5 \leq t \leq 5.01$	
$5 \leq t \leq 5.001$	

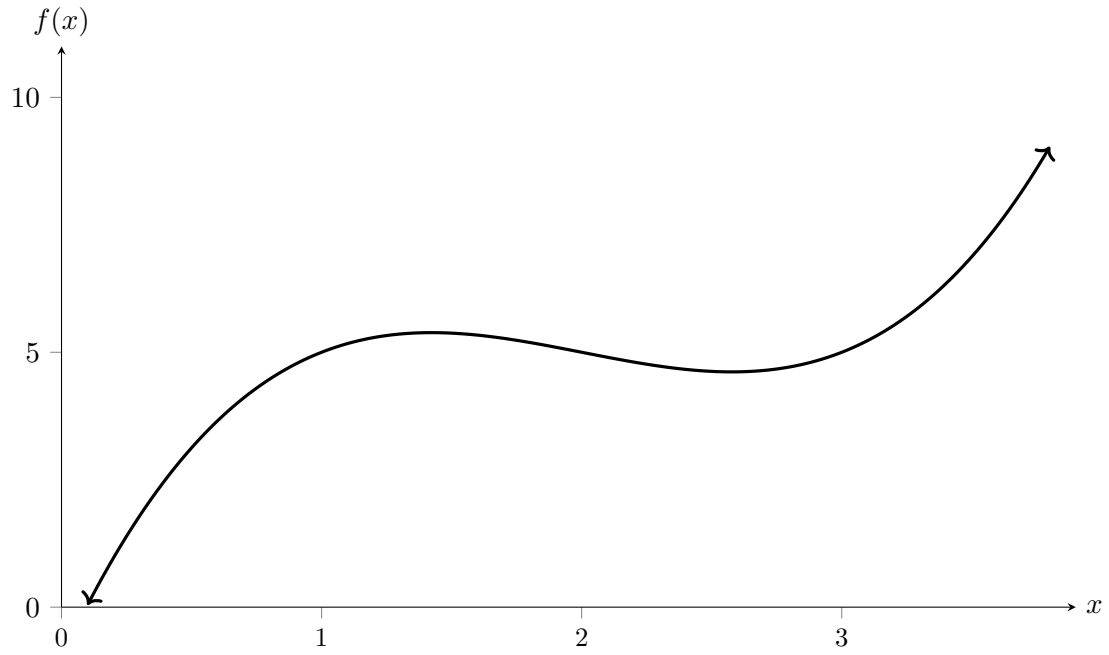
Question. Make a guess for what the velocity of the ball will be after *exactly* 5 seconds.

Question. Explain what is happening graphically in the above scenario.



The tangent and velocity problems – activity

Below is the graph of the function $f(x) = (x-2)^3 - x + 7$. The goal of this activity is to estimate the instantaneous change of $f(x)$ at $x = 1$.

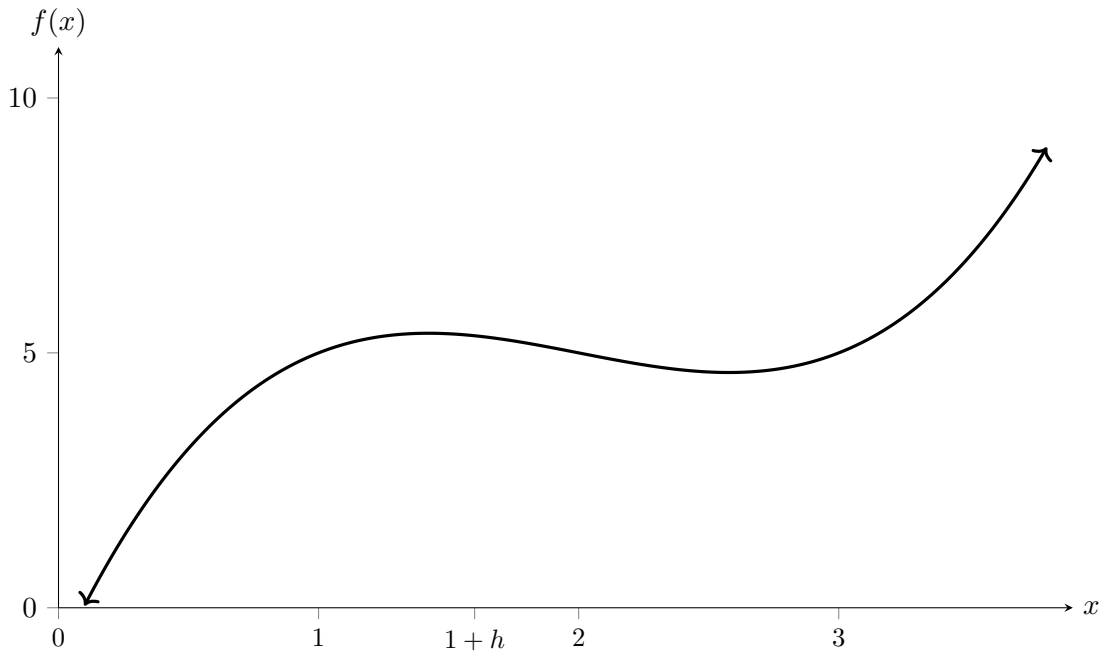


Question. Find the average rate of change of $f(x)$ on the interval $[1, 2]$. Draw the corresponding secant line on the graph above.

Question. Find the average rate of change of $f(x)$ on the interval $[1, 1.5]$. Draw the corresponding secant line on the graph above.

Question. Find the average rate of change of $f(x)$ on the interval $[1, 1.001]$. Draw the corresponding secant line on the graph above.

Question. Estimate the instantaneous change at $x = 1$. Draw the corresponding tangent line on the graph above.



Question. Let h be an arbitrary number that represents the distance between $x = 1$ and another x value. Find the average rate of change of $f(x)$ between $x = 1$ and $x = 1 + h$.

Question. Explain what we need to do to h in the expression above to estimate the instantaneous change at $x = 1$.

Question. Write down an expression that computes the instantaneous rate of change at an arbitrary value $x = a$.