Precalculus Review

1. Let $f(x) = \dfrac{5}{2x+1}$. Compute the difference quotient $\dfrac{f(a+h)-f(a)}{h}$ for $h \neq 0$.

Solution:

$$\begin{aligned}
\frac{f(a+h)-f(a)}{h} &= \frac{\dfrac{5}{2a+2h+1} - \dfrac{5}{2a+1}}{h} \\
&= \frac{5\big((2a+1)-(2a+2h+1)\big)}{h(2a+2h+1)(2a+1)} \\
&= \frac{-10h}{h(2a+2h+1)(2a+1)} \\
&= \frac{-10}{(2a+2h+1)(2a+1)}
\end{aligned}$$

2. Find an equation of the line through $(2,7)$ that is parallel to the line passing through $(1,3)$ and $(4,15)$.

Solution: Slope of the given line: $m = \dfrac{15-3}{4-1} = \dfrac{12}{3} = 4$. A parallel line through $(2,7)$ has slope 4:

$$y - 7 = 4(x-2) \implies \boxed{y = 4x - 1}.$$

3. Solve the equation $8x^2 - 2x - 3 = 0$ by factoring.

Solution:

$$8x^2 - 2x - 3 = (4x-3)(2x+1) = 0 \implies \boxed{x = \tfrac{3}{4} \quad \text{or} \quad x = -\tfrac{1}{2}}.$$

4. Find all real solutions to the equation $x^4 - 7x^2 + 10 = 0$.

Solution: Let $u = x^2$. Then $u^2 - 7u + 10 = (u-5)(u-2) = 0$, so

$$x^2 = 5 \quad \text{or} \quad x^2 = 2 \implies \boxed{x = \pm\sqrt{5},\ \pm\sqrt{2}}.$$

5. Factor the expression completely: $2x^3 + 5x^2 - 18x - 45$.

Solution: Group terms:

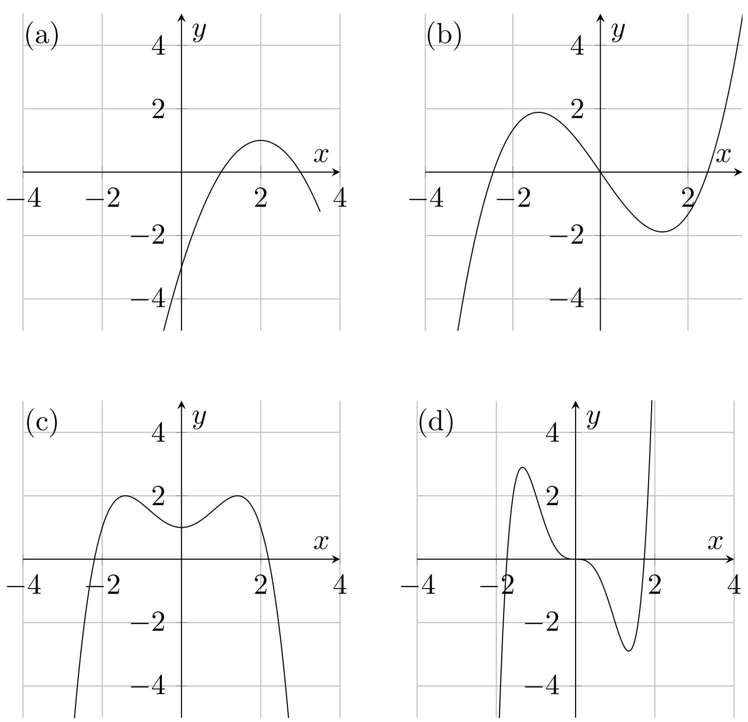
$$\begin{aligned} 2x^3 + 5x^2 - 18x - 45 &= (2x^3 + 5x^2) + (-18x - 45) \\ &= x^2(2x+5) - 9(2x+5) \\ &= (2x+5)(x^2-9) \\ &= (2x+5)(x-3)(x+3) \end{aligned}$$

6. Examine the leading term of $P(x) = -3x^4 + 6x^2 - 5x + 18$ and determine the far-left and far-right behavior of its graph.

Solution: The leading term is $-3x^4$: even degree with negative leading coefficient. Thus both ends go down:

$$\boxed{\lim_{x \to -\infty} P(x) = -\infty \quad \text{and} \quad \lim_{x \to +\infty} P(x) = -\infty}.$$

7. Match each graph to the corresponding polynomial.



(i) $f(x) = \frac{3}{2}x - 1$

(ii) $f(x) = -x^2 + 4x - 3$

(iii) $f(x) = \frac{1}{8}x^4 - x^2$

(iv) $f(x) = -\frac{1}{2}x^3 + x^2 + 1$

(v) $f(x) = x^4 - 2x^3 - x^2$

(vi) $f(x) = \frac{1}{3}x^3 - 2x$

(vii) $f(x) = -\frac{1}{4}x^4 + x^2 + 1$

(viii) $f(x) = x^5 - 3x^3$

Solution: Matchings:

$$\boxed{(a) \leftrightarrow \text{(ii)}, \quad (b) \leftrightarrow \text{(vi)}, \quad (c) \leftrightarrow \text{(vii)}, \quad (d) \leftrightarrow \text{(viii)}.}$$

(Quadratic opening down $\to$ (ii); odd cubic with origin symmetry $\to$ (vi); even quartic opening down with vertical shift $\to$ (vii); odd quintic with triple symmetry $\to$ (viii).)

8. Evaluate $\log_2\left(\frac{1}{16}\right)$.

Solution: $\frac{1}{16} = 2^{-4}$, so $\boxed{\log_2\left(\frac{1}{16}\right) = -4}$.

9. Evaluate $\sin(x)$, $\cos(x)$, $\tan(x)$, $\sec(x)$, $\csc(x)$, and $\cot(x)$ at $x = \dfrac{7\pi}{3}$.

Solution: $\dfrac{7\pi}{3} = 2\pi + \dfrac{\pi}{3}$, so it is coterminal with $\dfrac{\pi}{3}$ (Quadrant I).

$$\sin\left(\tfrac{7\pi}{3}\right) = \tfrac{\sqrt{3}}{2}, \quad \cos\left(\tfrac{7\pi}{3}\right) = \tfrac{1}{2}, \quad \tan\left(\tfrac{7\pi}{3}\right) = \sqrt{3}.$$

$$\sec\left(\tfrac{7\pi}{3}\right) = 2, \quad \csc\left(\tfrac{7\pi}{3}\right) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}, \quad \cot\left(\tfrac{7\pi}{3}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

10. Find the amplitude and period of $y = -3\sin(4\pi x)$. Then sketch the graph.

Solution: In $y = A\sin(Bx)$, amplitude $= |A| = 3$. Here $B = 4\pi$, so the period is

$$T = \frac{2\pi}{B} = \frac{2\pi}{4\pi} = \boxed{\tfrac{1}{2}}.$$

The negative sign reflects the sine across the x-axis.