

MATH 1150: Midterm 2 Practice Solutions

1. Find the inverse function $g^{-1}(x)$ of the function:

$$g(x) = \frac{5}{x+4}.$$

Solution: Let $y = \frac{5}{x+4}$. Interchange x and y to obtain

$$x = \frac{5}{y+4}.$$

Multiplying both sides by $(y+4)$ gives

$$x(y+4) = 5.$$

Solving for y , we have

$$y+4 = \frac{5}{x} \implies y = \frac{5}{x} - 4.$$

Thus,

$$g^{-1}(x) = \frac{5}{x} - 4.$$

2. The function $q(x)$ is given by the table:

x	$q(x)$
-4	5
-2	3
1	0
3	-1
5	4

Find $q^{-1}(3)$, if it exists.

Solution: We look for the value of x such that $q(x) = 3$. From the table, we see that $q(-2) = 3$. Hence,

$$q^{-1}(3) = -2.$$

3. A line passes through the point $(3, 1)$ and is perpendicular to the line

$$y = -\frac{3}{4}x + 2.$$

Find the equation of the new line.

Solution: The slope of the given line is $-\frac{3}{4}$. The slope of any line perpendicular to it is the negative reciprocal:

$$m = \frac{4}{3}.$$

Using the point-slope form, we have:

$$y - 1 = \frac{4}{3}(x - 3).$$

Thus, the equation of the line is

$$y = \frac{4}{3}(x - 3) + 1 = \frac{4}{3}x - 4 + 1 = \frac{4}{3}x - 3.$$

4. Find the equation of a circle whose center is at $(6, -2)$ and has a diameter of 10 units.

Solution: The radius is half the diameter:

$$r = \frac{10}{2} = 5.$$

The standard form of a circle with center (h, k) and radius r is:

$$(x - h)^2 + (y - k)^2 = r^2.$$

Therefore, the equation is:

$$(x - 6)^2 + (y + 2)^2 = 25.$$

5. Solve for x in the equation:

$$4(x + 1)^2 - 9 = 19.$$

Solution: Add 9 to both sides:

$$4(x + 1)^2 = 28.$$

Divide by 4:

$$(x + 1)^2 = 7.$$

Taking the square root gives:

$$x + 1 = \pm\sqrt{7},$$

so that

$$x = -1 \pm \sqrt{7}.$$

6. Solve for x in the equation:

$$x^4 - 8x^2 + 16 = 0.$$

Solution: Let $u = x^2$. Then the equation becomes:

$$u^2 - 8u + 16 = 0.$$

This factors as:

$$(u - 4)^2 = 0,$$

so that $u = 4$. Substituting back $x^2 = 4$, we get:

$$x = \pm 2.$$

7. Identify the domain and range of the exponential function

$$h(x) = 3 - 2^x.$$

Solution: Since 2^x is defined for all real numbers, the domain of $h(x)$ is

$$(-\infty, \infty).$$

Because $2^x > 0$ for all x , we have $3 - 2^x < 3$. As $x \rightarrow -\infty$, $2^x \rightarrow 0$, so $h(x) \rightarrow 3$. As $x \rightarrow \infty$, $2^x \rightarrow \infty$, so $h(x) \rightarrow -\infty$. Therefore, the range is:

$$(-\infty, 3).$$

8. Identify the domain and range of the logarithmic function

$$k(x) = \log_3(x - 7) + 5.$$

Solution: The argument of the logarithm must be positive:

$$x - 7 > 0 \implies x > 7.$$

Thus, the domain is

$$(7, \infty).$$

Since logarithmic functions can take any real value, the range is:

$$(-\infty, \infty).$$

9. Compute

$$\log_5 \left(\frac{1}{25} \right), \quad \log_5(1), \quad \log_5(25).$$

Solution: Recognize that $25 = 5^2$ and $\frac{1}{25} = 5^{-2}$. Hence,

$$\log_5 \left(\frac{1}{25} \right) = \log_5(5^{-2}) = -2,$$

$$\log_5(1) = 0,$$

$$\log_5(25) = \log_5(5^2) = 2.$$

10. Given

$$f(x) = \frac{1}{4} \log_6(x + 5),$$

find its inverse function $f^{-1}(x)$.

Solution: Let $y = \frac{1}{4} \log_6(x + 5)$. Swap x and y :

$$x = \frac{1}{4} \log_6(y + 5).$$

Multiply both sides by 4:

$$4x = \log_6(y + 5).$$

Convert to exponential form:

$$y + 5 = 6^{4x}.$$

Thus,

$$y = 6^{4x} - 5,$$

and the inverse function is:

$$f^{-1}(x) = 6^{4x} - 5.$$

11. A tank is being drained at a constant rate of 4 liters per minute. Initially, it contains 100 liters of water.
- (a) Construct a linear model $V(t)$ for the volume of water remaining after t minutes.
 - (b) Determine how long it takes for the tank to be empty.

Solution:

- (a) The volume function is given by:

$$V(t) = 100 - 4t.$$

- (b) To find when the tank is empty, set $V(t) = 0$:

$$100 - 4t = 0 \implies t = 25.$$

Thus, the tank will be empty after 25 minutes.

12. A toy rocket is launched from the ground, and its height in meters after t seconds is given by:

$$h(t) = -4(t - 2)^2 + 50.$$

- (a) Find the maximum height achieved by the rocket.
- (b) Determine when the rocket lands back on the ground.

Solution:

- (a) The height function is in vertex form. The vertex occurs at $t = 2$, with $h(2) = 50$, so the maximum height is 50 meters.

- (b) To determine when the rocket lands, set $h(t) = 0$:

$$-4(t - 2)^2 + 50 = 0 \implies -4(t - 2)^2 = -50,$$

$$(t - 2)^2 = \frac{50}{4}$$

Taking the square root yields:

$$t - 2 = \pm \frac{5\sqrt{2}}{2}.$$

Discarding the negative solution (since time must be positive), we have:

$$t = 2 + \frac{5\sqrt{2}}{2}.$$

Thus, the rocket lands at $t = 2 + \frac{5\sqrt{2}}{2}$ seconds.

13. Describe the end behavior of the polynomial function:

$$g(x) = -(x + 3)^3(x - 5)^2(x + 1).$$

Solution: The total degree of the polynomial is $3 + 2 + 1 = 6$ (an even degree). The leading coefficient, after expanding, is negative. Therefore, as $x \rightarrow \pm\infty$,

$$g(x) \rightarrow -\infty.$$

14. Consider the rational function:

$$r(x) = \frac{2(x-2)(x+3)(x+1)^2(x+5)^2}{5(x+3)^2(x+1)(x-4)^3}.$$

- (a) Identify the vertical asymptotes.
- (b) Determine whether a horizontal asymptote exists.
- (c) Locate any holes in the function.

Solution: The simplified function is:

$$r(x) = \frac{(x-2)(x+1)(x+5)^2}{(x+3)(x-4)^3}.$$

(a) Vertical Asymptotes: Vertical asymptotes occur where the denominator is zero (and the factor is not canceled completely). In the simplified form, the denominator is

$$5(x+3)(x-4)^3.$$

Setting each factor equal to zero gives:

$$x+3=0 \implies x=-3,$$

$$x-4=0 \implies x=4.$$

Thus, the vertical asymptotes are at $x = -3$ and $x = 4$.

(b) Horizontal Asymptote: To find the horizontal asymptote, we compare the degrees of the numerator and the denominator. In the simplified function, the degrees are equal, so the horizontal asymptote is the ratio of the leading coefficients.

$$y = \frac{2}{5}.$$

(c) Holes: Holes occur where a common factor cancels between the numerator and denominator. In the original function, the factor $(x+1)$ appears as $(x+1)^2$ in the numerator and $(x+1)$ in the denominator; after cancellation, one factor of $(x+1)$ remains in the numerator. This cancellation creates a hole at the value

$$x = -1.$$

To find the y -coordinate of the hole, substitute $x = -1$ into the simplified function:

$$r(-1) = \frac{(-3)(0)(16)}{2(-125)} = 0.$$

Therefore, the hole is at $(-1, 0)$.

15. A scientist observes that the population of bacteria in a petri dish doubles every hour. Initially, there are 20 bacteria.
- (a) Express the population as a function of time t in hours.
 - (b) Determine how many bacteria there will be after 5 hours.

Solution:

(a) Since the bacteria double every hour, the population grows exponentially. The general formula for exponential growth is:

$$P(t) = P_0 \cdot 2^t,$$

where P_0 is the initial population. Given that $P_0 = 20$, the population function is:

$$P(t) = 20 \cdot 2^t.$$

(b) To find the number of bacteria after 5 hours, substitute $t = 5$ into the function:

$$P(5) = 20 \cdot 2^5 = 20 \cdot 32 = 640.$$

Thus, after 5 hours, there will be 640 bacteria.