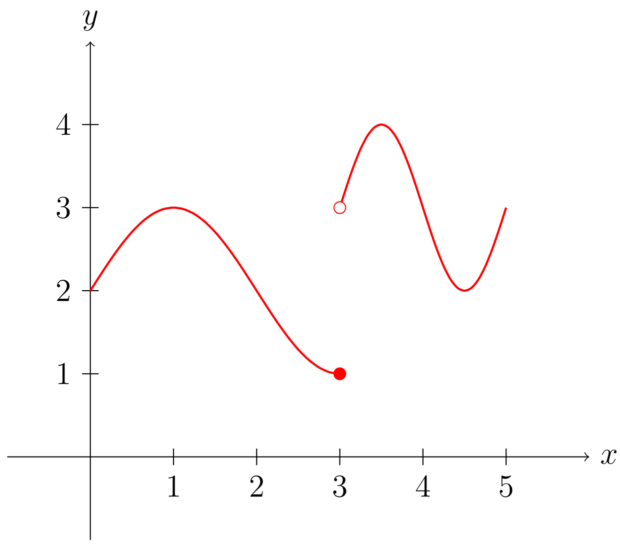
IC 28: Limits

1. The graph of a function f is shown below.



Answer the following:

(a) $\lim_{x\to 1} f(x) = \boxed{3}$

(b) $\lim_{x\to 3^-} f(x) = \boxed{1}$

(c) $\lim_{x\to 3^+} f(x) = \boxed{3}$

(d) $\lim_{x\to 3} f(x) = \boxed{\text{DNE}}$

(e) $f(3) = \boxed{1}$

2. Evaluate $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$.

Solution. First, factor the numerator:

$$\frac{x^2 + x - 6}{x - 2} = \frac{(x-2)(x+3)}{x-2}$$

For $x \neq 2$, cancel:

$$= x + 3$$

Now take the limit:

$$\lim_{x \to 2}(x+3) = 2 + 3 = \boxed{5}$$

3. Evaluate $\lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}$.

Solution. Multiply numerator and denominator by the conjugate:

$$\frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2} = \frac{(4+h) - 4}{h(\sqrt{4+h} + 2)} = \frac{h}{h(\sqrt{4+h} + 2)}$$

Simplify:

$$= \frac{1}{\sqrt{4+h} + 2}$$

Now take the limit as $h \to 0$:

$$= \frac{1}{\sqrt{4+0} + 2} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$$