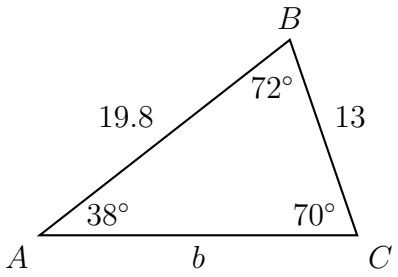
IC25: Laws of Sines and Cosines — Solutions

1. Use the Law of Sines to find the side c.
   Given: $\angle A = 38°$, $\angle B = 72°$, $a = 13$

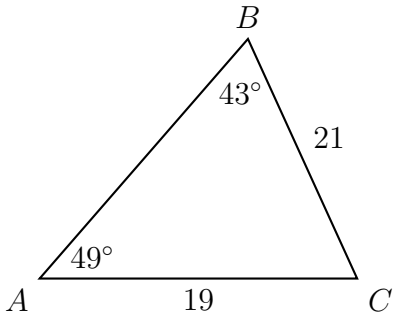


   Solution. We find angle C and use the Law of Sines:

$$\angle C = 180° - 38° - 72° = 70°$$

$$c = \frac{a \cdot \sin C}{\sin A} = \frac{13 \cdot \sin 70°}{\sin 38°} \approx \frac{13 \cdot 0.9397}{0.6157} \approx \boxed{19.8}$$

2. Use the Law of Sines to find the angle $\angle B$.
   Given: $a = 21$, $b = 19$, $\angle A = 49°$

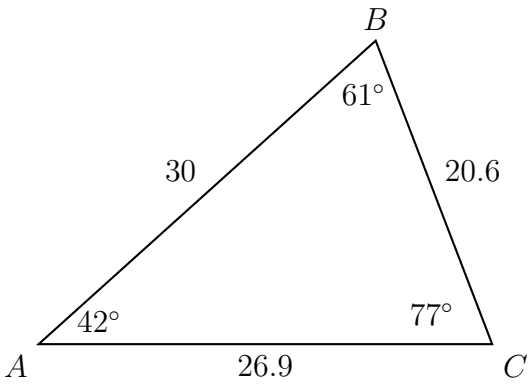


   Solution. Use the Law of Sines:

$$\sin B = b \cdot \frac{\sin A}{a} = 19 \cdot \frac{\sin 49°}{21} \approx 19 \cdot 0.0359 \approx 0.6822 \Rightarrow \angle B = \arcsin(0.6822) \approx \boxed{43°}$$

   Why only one triangle? The other possibility of $180° - 43° = 137°$, would result in a total angle measure that exceeds 180°. So only one triangle is valid.

3. Solve triangle ABC given: $\angle A = 42°$, $\angle B = 61°$, $c = 30$



Solution.

$$\angle C = 180° - 42° - 61° = 77°$$

$$a = \frac{30 \cdot \sin 42°}{\sin 77°} \approx \boxed{20.6}, \quad b = \frac{30 \cdot \sin 61°}{\sin 77°} \approx \boxed{26.9}$$

4. Solve triangle ABC given: $a = 15$, $c = 22$, and $\angle A = 38°$. If two triangles exist, find both.

Step 1: Use the Law of Sines to solve for $\angle C$

$$\frac{\sin C}{c} = \frac{\sin A}{a} \quad \Rightarrow \quad \sin C = c \cdot \frac{\sin A}{a} = 22 \cdot \frac{\sin 38°}{15} \approx 22 \cdot \frac{0.6157}{15} \approx 0.902$$

$$\Rightarrow \angle C_1 = \arcsin(0.902) \approx \boxed{64.4°}, \quad \angle C_2 = 180° - 64.4° = \boxed{115.6°}$$

Step 2: Compute $\angle B$ in both cases

First triangle:

$$\angle B_1 = 180° - 38° - 64.4° = \boxed{77.6°}$$

Second triangle:

$$\angle B_2 = 180° - 38° - 115.6° = \boxed{26.4°}$$

Step 3: Solve for side b in both cases using the Law of Sines

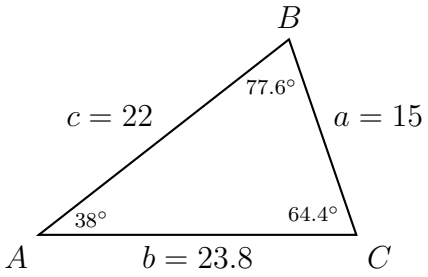
First triangle:

$$b = \frac{a \cdot \sin B}{\sin A} = \frac{15 \cdot \sin 77.6°}{\sin 38°} \approx \frac{15 \cdot 0.9786}{0.6157} \approx \boxed{23.8}$$

Second triangle:

$$b = \frac{15 \cdot \sin 26.4°}{\sin 38°} \approx \frac{15 \cdot 0.4446}{0.6157} \approx \boxed{10.8}$$

First triangle:



Second triangle: