IC 12 - Functions So Far Solutions

Linear Function Solution: A taxi service charges a flat fee of \$2.50 plus \$1.75 per mile.

(a) Model the Situation. A linear function in slope-intercept form is given by

$$\boxed{C(m) = 1.75m + 2.50.}$$

(b) Calculate the Cost for a 10-Mile Ride. Substitute $m = 10$ into the function:

$$C(10) = 1.75(10) + 2.50 = 17.50 + 2.50 = \boxed{\$20.00.}$$

Quadratic Function Solution: A ball is thrown upward from the top of a 48-foot building with an initial velocity of 32 ft/s. Its height is given by

$$h(t) = -16t^2 + 32t + 48.$$

(a) Time of Maximum Height. For a quadratic function $h(t) = at^2 + bt + c$, the vertex occurs at

$$t = -\frac{b}{2a}.$$

Here, $a = -16$ and $b = 32$, so:

$$t = -\frac{32}{2(-16)} = -\frac{32}{-32} = \boxed{1 \text{ second.}}$$

(b) Maximum Height. Substitute $t = 1$ into $h(t)$:

$$h(1) = -16(1)^2 + 32(1) + 48 = -16 + 32 + 48 = \boxed{64 \text{ feet.}}$$

(c) When the Ball Hits the Ground. Set $h(t) = 0$:

$$\begin{aligned} -16t^2 + 32t + 48 &= 0 \\ 16t^2 - 32t - 48 &= 0 \\ t^2 - 2t - 3 &= 0 \\ (t-3)(t+1) &= 0 \end{aligned}$$

Thus, $t = 3$ or $t = -1$. Since time cannot be negative, the ball hits the ground at:

$$\boxed{t = 3 \text{ seconds.}}$$

Polynomial Function Solution. Consider the polynomial

$$f(x) = \frac{1}{1000}(x+3)(x-2)^2(x-9)^3.$$

(a) **Zeros and Their Multiplicities.**
The zeros are found by setting each factor equal to zero:

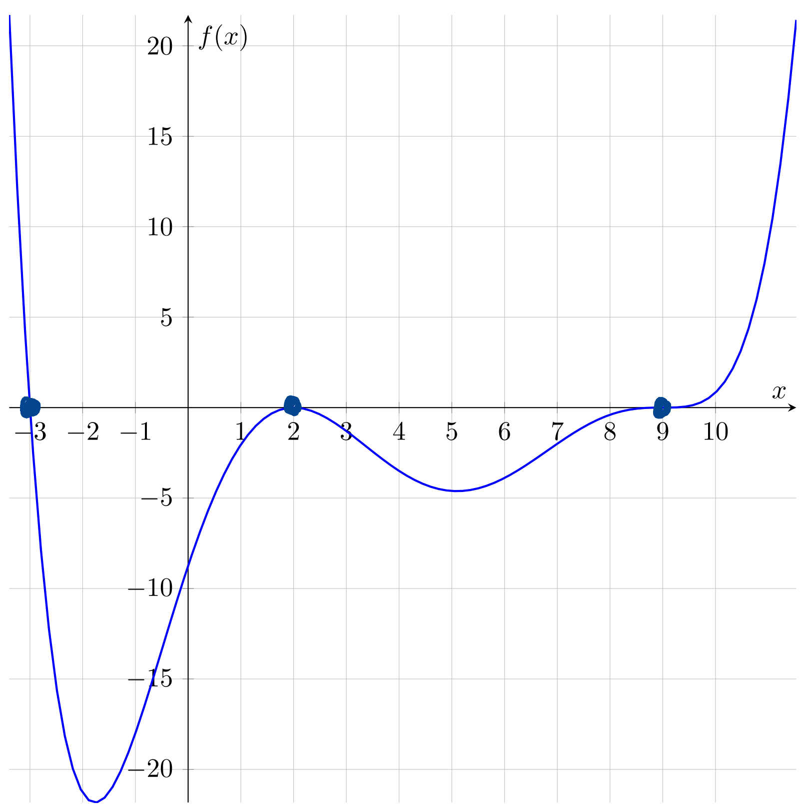
$$\begin{aligned} x+3=0 &\quad \Rightarrow x=-3 \quad \text{(multiplicity 1)},\\ x-2=0 &\quad \Rightarrow x=2 \quad \text{(multiplicity 2)},\\ x-9=0 &\quad \Rightarrow x=9 \quad \text{(multiplicity 3)}. \end{aligned}$$

(b) **Sign Chart and End Behavior.**
The degree of $f(x)$ is $1+2+3=6$ (even) and the leading coefficient is positive. Thus, as $x \to \pm\infty$, $f(x) \to +\infty$.

We now analyze the sign of each factor on the intervals determined by the zeros:

Interval	$\frac{1}{1000}$	$x+3$	$(x-2)^2$	$(x-9)^3$	$f(x)$
$x<-3$	Positive	Negative	Positive	Negative	$(+)(-)(+)(-)=+$
$-3<x<2$	Positive	Positive	Positive	Negative	$(+)(+)(+)(-)=-$
$2<x<9$	Positive	Positive	Positive	Negative	$(+)(+)(+)(-)=-$
$x>9$	Positive	Positive	Positive	Positive	$(+)(+)(+)(+)=+$

(c) **Sketch of the Graph.**



Rational Function Solution. Analyze the rational function

$$R(x) = \frac{(x-5)(x-3)(x+2)}{(x-3)(x+1)(x+4)}.$$

Simplification: Notice the common factor $(x-3)$ in the numerator and denominator. Canceling it (with the understanding that $x \neq 3$) yields:

$$R(x) = \frac{(x-5)(x+2)}{(x+1)(x+4)} \quad \text{for } x \neq 3.$$

(a) **x- and y-Intercepts.**

- x-intercepts: Set the numerator equal to zero:

$$(x-5)(x+2) = 0 \quad \Rightarrow \quad x = 5 \quad \text{or} \quad x = -2.$$

- y-intercept: Substitute $x = 0$:

$$R(0) = \frac{(0-5)(0+2)}{(0+1)(0+4)} = \frac{(-5)(2)}{(1)(4)} = \frac{-10}{4} = -2.5.$$

(b) **Holes.**
The canceled factor $(x-3)$ indicates a removable discontinuity (hole) at $x = 3$. To find the y-value of the hole, substitute $x = 3$ into the simplified function:

$$R(3) = \frac{(3-5)(3+2)}{(3+1)(3+4)} = \frac{(-2)(5)}{(4)(7)} = \frac{-10}{28} = -\frac{5}{14}.$$

Hence, there is a hole at $\left(3, -\frac{5}{14}\right)$.

(c) **Vertical Asymptotes.**
Set the denominator of the simplified function equal to zero:

$$(x+1)(x+4) = 0 \quad \Rightarrow \quad x = -1 \quad \text{or} \quad x = -4.$$

Thus, the vertical asymptotes are $x = -1$ and $x = -4$.

(d) **Horizontal Asymptote.**
The numerator and denominator of the simplified function are both quadratic (degree 2). Therefore, the horizontal asymptote is the ratio of their leading coefficients:

$$y = \frac{1}{1} = 1.$$

(e) **Sketch of the Graph.**

To determine the sign of $R(x)$, we analyze the factors of the simplified function:

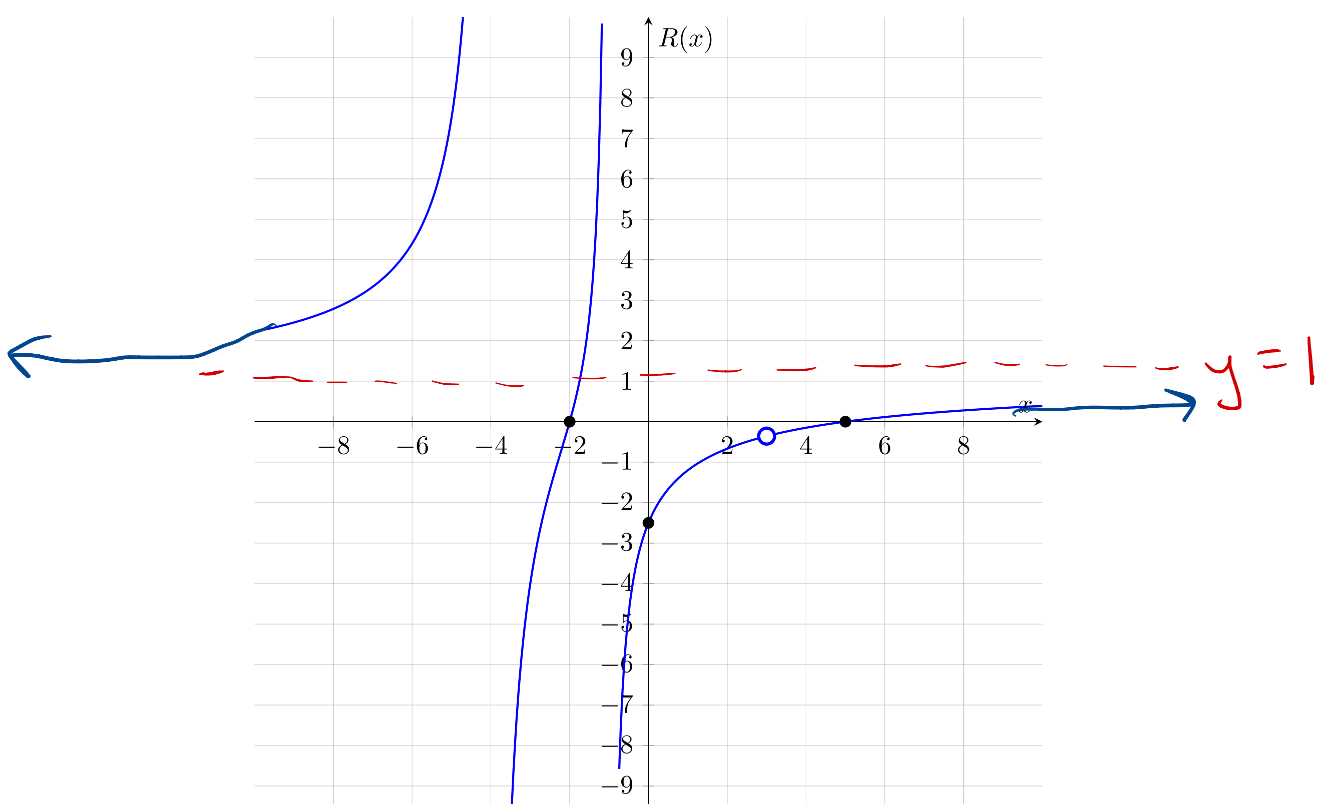
$$R(x) = \frac{(x-5)(x+2)}{(x+1)(x+4)}.$$

The critical points from the factors are:

$$x = -4, \quad x = -2, \quad x = -1, \quad x = 5.$$

(These points are where ~~to~~ either the numerator or the denominator is 0.)

Interval	$x+2$	$x-5$	$x+1$	$x+4$	**Overall Sign**
$x < -4$	$-$	$-$	$-$	$-$	$+$
$-4 < x < -2$	$-$	$-$	$-$	$+$	$-$
$-2 < x < -1$	$+$	$-$	$-$	$+$	$+$
$-1 < x < 5$	$+$	$-$	$+$	$+$	$-$
$x > 5$	$+$	$+$	$+$	$+$	$+$



Exponential Function Solution. Suppose that \$1000 is invested at an annual interest rate of 5% for 10 years. Calculate the future value using the following compounding methods.

Let $P = 1000$, $r = 0.05$, and $t = 10$.

(a) Annually.

$$A = 1000(1.05)^{10} \approx \boxed{\$1628.89.}$$

(b) Semiannually.

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{0.05}{2}\right)^{2\times 10} = 1000(1.025)^{20} \approx \boxed{\$1638.62.}$$

(c) Quarterly.

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{0.05}{4}\right)^{4\times 10} = 1000(1.0125)^{40} \approx \boxed{\$1644.00.}$$

(d) Continuously.

$$A = Pe^{rt} = 1000e^{0.05\times 10} = 1000e^{0.5} \approx \boxed{\$1648.72.}$$