

Math 1150: Final Exam Practice

Radical & Exponential Simplification

1. Simplify the expression. Assume all variables are positive $\left(\frac{81a^6}{b^{12}}\right)^{1/2}$.
2. Simplify $\left(\frac{27p^3q^{-6}}{3p^{-2}q^2}\right)^{1/3}$.

Inequalities

1. Solve the inequality $|3x - 4| > 5$.
2. Solve the inequality $|-4x + 8| \leq 12$.
3. Solve $(x + 1)(x - 7) < 0$.
4. Solve $(x - 2)(x + 5) < 0$.

Difference Quotients

1. Compute the difference quotient $\frac{f(x+h) - f(x)}{h}$ for $f(x) = 2x^2 + 5x$.
2. Compute the difference quotient $\frac{f(x+h) - f(x)}{h}$ for $f(x) = 3x^2 - 5x + 1$.

Polynomial Equations

1. Solve for x : $(x - 4)(x + 6) = 5$.
2. Solve for x : $(2x - 3)(x + 4) = 6$.

Domain & Rational Functions

1. Determine the domain of $f(x) = \frac{\sqrt{x+1}}{x^2 - 9}$.
2. Determine the domain of $f(x) = \frac{\sqrt{5x+4}}{x^2 - 1}$.
3. Identify all vertical asymptotes of $f(x) = \frac{\sqrt{x-3}}{x^2 - 4x + 3}$. (Challenge)
4. Analyze the end behavior of $f(x) = \frac{5x^3 - x + 1}{x^3 + 2x - 4}$.
5. Analyze the end behavior of $f(x) = \frac{2x^2 - 7x + 4}{x^3 + 5x - 2}$.

Function Operations & Inverses

1. Given $f(-3) = 4$, $f(0) = 2$, $f(2) = -1$ and $g(x) = 2x^2 - 1$, find $g(f(0))$.
2. Given $f(0) = 3$, $f(2) = 5$, $f(4) = -1$ and $g(x) = x + 2$, find $f(g(2))$.
3. Describe the sequence of transformations taking $f(x) = x^3$ to $h(x) = -2(x+1)^3 - 4$, then find $h^{-1}(y)$.
4. Evaluate $f^{-1}(2)$ if $f(1) = 4$, $f(2) = 2$, $f(3) = -3$, $f(4) = 1$.
5. Find the inverse of $f(x) = 3x - 7$.

Lines, Parabolas & Circles

1. Find the equation of the line perpendicular to $y = -\frac{1}{2}x + 3$ passing through $(2, -1)$.
2. Write the equation of the line parallel to $y = 7x - 2$ passing through $(-1, 3)$.
3. Find the vertex of the parabola $f(x) = -x^2 + 6x - 2$.
4. Identify the center and radius of the circle $(x+5)^2 + (y-3)^2 = 36$.

Optimization Problems

1. A rectangular pen is built next to a barn wall using 40 ft of fencing (two sides length x , one side y).
 - (a) Express the area in terms of x and y .
 - (b) Write the area as a function of x alone.
 - (c) Find the values of x and y that maximize the area.
2. Find the maximum area of a rectangle with perimeter 50. Give its dimensions.
3. A revenue model is $R(p) = -5p^2 + 100p$.
 - (a) Find the price p that maximizes revenue.
 - (b) Compute the maximum revenue.

Logarithmic & Exponential Functions

1. Evaluate $\log_4\left(\frac{1}{64}\right)$.
2. Simplify $\log_2(16) + \log_2\left(\frac{1}{2}\right)$.
3. Convert $\ln(y) = 7$ into exponential form.
4. Convert $\ln(8) = y$ into exponential form.
5. Solve $\log_3(x^2 + 1) = 2$.
6. Solve $\log_2(x^2 - 4) = 3$. State any restrictions.
7. Evaluate $\log_3(9) - \log_3\left(\frac{1}{3}\right)$.

Trigonometric Evaluation & Identities

1. Evaluate $\tan\left(\frac{5\pi}{6}\right)$.
2. Simplify $\frac{\sin^2(x) + \cos^2(x)}{\tan(x)}$.
3. Simplify $\frac{1 - \cos(2x)}{\sin(x)}$.
4. Evaluate $\sin\left(\frac{7\pi}{3}\right)$ by finding a coterminal angle.
5. Use a sum identity to evaluate $\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$.
6. Use a sum identity to compute $\sin\left(\frac{5\pi}{12}\right)$.
7. Use a half-angle identity to evaluate $\sin\left(\frac{5\pi}{8}\right)$.
8. Given $\cos(\theta) = \frac{4}{5}$ with $0 < \theta < \frac{\pi}{2}$, find $\cos(2\theta)$.
9. Given $\sin(\phi) = \frac{7}{25}$ with $\frac{\pi}{2} < \phi < \pi$, find $\cos\left(\frac{\phi}{2}\right)$.
10. Given $\cos(\theta) = \frac{5}{13}$ with $0 < \theta < \frac{\pi}{2}$, find $\cos(2\theta)$.

Inverse Trigonometric Functions

1. Evaluate $\arcsin(-1)$.
2. Evaluate $\arccos\left(-\frac{1}{2}\right)$.
3. Evaluate $\tan^{-1}(1)$ and explain the result.
4. Evaluate $\cos^{-1}\left(\cos\left(\frac{11\pi}{6}\right)\right)$.

Trigonometric Equations

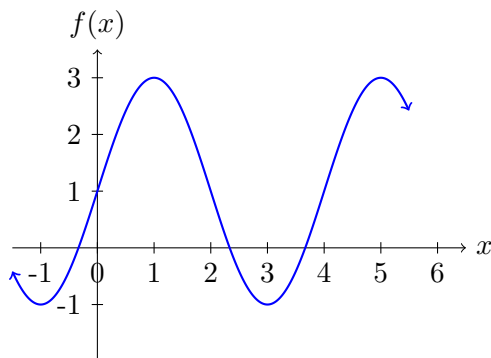
1. Find all solutions to $2\cos(3x) = 1$ for $0 \leq x \leq 2\pi$.
2. Find all solutions to $\cos^2(x) - \sin(x) = 1$ in $[0, 4\pi]$.
3. Find all solutions to $2\sin(x) = \sqrt{3}$ in $[0, 2\pi]$.
4. Solve $\sin^2(x) + \sin(x) = 0$ for all real x .

Triangle Applications

1. A 12-ft ladder leans against a wall, making a 60° angle with the ground. How high up the wall does it reach?
2. In a triangle with angles 30° , 60° and hypotenuse 8, find the side opposite 30° .
3. In a right triangle with hypotenuse 10 and one angle 30° , find the side opposite 30° .
4. In triangle ABC, $a=6$, $b=7$, $\angle C = 45^\circ$. Find c via the Law of Cosines.
5. In a triangle with $a=5$, $b=7$, $\angle C = 60^\circ$, use the Law of Cosines to find c .

Graphing & Transformations

1. State the amplitude, period, and midline of $f(x) = 3 - 4\cos(2x)$.
2. Sketch one period of $y = 3\cos(\pi x) - 2$, labeling key points.
3. What is the equation of the function whose graph is shown below?



4. Sketch a function f satisfying:
 - $\lim_{x \rightarrow 1^-} f(x) = -\infty$
 - $\lim_{x \rightarrow 1^+} f(x) = \infty$
 - $f(0) = 0, f(2) = 2$
 - Horizontal asymptote $y = 1$
5. Sketch the graph of a function $g(x)$ satisfying all of the following properties:
 - As $x \rightarrow 2^-$, $g(x) \rightarrow 5$.
 - As $x \rightarrow 2^+$, $g(x) \rightarrow -3$.
 - $g(2) = 3$
 - $g(0) = 1$
 - $\lim_{x \rightarrow -1} g(x) = 2$ but $g(-1) = 0$
 - As $x \rightarrow +\infty$, $g(x) \rightarrow 0$.
 - As $x \rightarrow -\infty$, $g(x) \rightarrow 1$.