Math 1150: Midterm 3 Practice Solutions

1. Simplify the expression
$$\log_2(48) - \log_2(3).$$

Solution: Combine the logs to get
$$\log_2\left(\frac{48}{3}\right) = \log_2(16) = 4.$$

2. Solve for x:
$$\log_3(2x-4) + \log_3(x+1) = 2.$$

Solution: Combine the logarithms:
$$\log_3[(2x-4)(x+1)] = 2.$$
Therefore, $(2x-4)(x+1) = 3^2 = 9$. Expanding,
$$2x^2 - 2x - 4 = 9 \quad \Longrightarrow \quad 2x^2 - 2x - 13 = 0.$$
Solve using the quadratic formula:
$$x = \frac{2 \pm \sqrt{4+104}}{4} = \frac{2 \pm \sqrt{108}}{4} = \frac{2 \pm 6\sqrt{3}}{4} = \frac{1 \pm 3\sqrt{3}}{2}.$$
Check the domain: $2x - 4 > 0$ requires $x > 2$. Only $x = \frac{1+3\sqrt{3}}{2}$ is valid.

3. Solve for x:

$$\log_5(3x-4) = \log_5(x+2) + 1.$$

Solution: First, subtract $\log_5(x+2)$ from both sides:

$$\log_5(3x-4) - \log_5(x+2) = 1.$$

By the quotient property of logarithms, this becomes:

$$\log_5\left(\frac{3x-4}{x+2}\right) = 1.$$

Using the definition of logarithm, we set the argument equal to 5^1:

$$\frac{3x-4}{x+2} = 5.$$

Multiply both sides by $x+2$:

$$3x-4 = 5(x+2).$$

Expanding and simplifying gives:

$$3x-4 = 5x+10 \quad\Longrightarrow\quad -2x = 14 \quad\Longrightarrow\quad x = -7.$$

Now, check the domain restrictions. The expressions $\log_5(3x-4)$ and $\log_5(x+2)$ require that

$$3x-4 > 0 \quad \text{and} \quad x+2 > 0.$$

For $x = -7$:

$$3(-7) - 4 = -25 < 0 \quad \text{and} \quad -7+2 = -5 < 0.$$

Since $x = -7$ does not satisfy the domain conditions, there is no valid solution.

4. Expand the expression completely:

$$\ln\left(\frac{(2x+3)^{3/2}}{\sqrt{x}}\right).$$

Solution: Write the square roots as exponents:

$$\ln\left(\frac{(2x+3)^{3/2}}{x^{1/2}}\right) = \frac{3}{2}\ln(2x+3) - \frac{1}{2}\ln x.$$

This is the expanded form.

5. Solve the equation for x:

$$e^{2x} - 7e^x + 10 = 0.$$

Solution: Let $u = e^x$, so the equation becomes

$$u^2 - 7u + 10 = 0.$$

Factor:

$$(u-5)(u-2) = 0 \quad \Longrightarrow \quad u = 5 \text{ or } u = 2.$$

Therefore, $e^x = 5$ or $e^x = 2$, giving

$$x = \ln 5 \quad \text{or} \quad x = \ln 2.$$

6. A bacteria culture grows exponentially, increasing from 150 to 600 individuals in 3 hours. Write an exponential growth function for the population and determine the predicted population after 5 hours.

Solution: Let $P(t) = P_0 e^{kt}$ with $P_0 = 150$. Given $P(3) = 600$,

$$150e^{3k} = 600 \quad \Longrightarrow \quad e^{3k} = 4 \quad \Longrightarrow \quad k = \frac{\ln 4}{3}.$$

The function is

$$P(t) = 150\, e^{\frac{\ln 4}{3} t}.$$

After 5 hours:

$$P(5) = 150\, e^{\frac{5 \ln 4}{3}} = 150 \cdot 4^{5/3}.$$

7. Convert $135°$ to radians.

Solution: Multiply by $\frac{\pi}{180}$:

$$135° \times \frac{\pi}{180} = \frac{3\pi}{4} \text{ radians.}$$

8. Find the degree measure of an angle whose radian measure is $\frac{9\pi}{10}$.

Solution: Multiply by $\frac{180}{\pi}$:

$$\frac{9\pi}{10} \times \frac{180}{\pi} = 162^\circ.$$

9. Determine all coterminal angles of $\theta = 2$ radians that lie between -2π and 2π.

Solution: The general form is $2 + 2\pi k$ for $k \in \mathbb{Z}$. Within $-2\pi \leq \theta \leq 2\pi$:

$$k = 0 : \quad 2,$$

$$k = -1 : \quad 2 - 2\pi.$$

For $k = 1$, $2 + 2\pi > 2\pi$. Thus, the coterminal angles are 2 and $2 - 2\pi$.

10. Find the reference angle for 245°.

Solution: 245° is in the third quadrant. Its reference angle is

$$245^\circ - 180^\circ = 65^\circ.$$

11. Find the reference angle for -170°.

Solution: Find a positive coterminal angle: $-170^\circ + 360^\circ = 190^\circ$. Then,

$$190^\circ - 180^\circ = 10^\circ.$$

12. The terminal side of an angle in standard position passes through the point $(-8, -6)$. Find $\sin(\theta)$ and $\cos(\theta)$.

Solution: Compute the radius:

$$r = \sqrt{(-8)^2 + (-6)^2} = \sqrt{64 + 36} = 10.$$

Thus,

$$\sin\theta = \frac{-6}{10} = -\frac{3}{5}, \quad \cos\theta = \frac{-8}{10} = -\frac{4}{5}.$$

13. A circular garden has a radius of 30 meters. If a pathway covers one-third of the circle's circumference, what is the length of the pathway?

Solution: The full circumference is

$$2\pi(30) = 60\pi.$$

One-third of this is

$$\frac{1}{3} \cdot 60\pi = 20\pi \text{ meters.}$$

14. Simplify the following expression:

$$\frac{1 - \sin^2 x}{\sin x \cos x}.$$

Solution: Use $\sin^2 x + \cos^2 x = 1$ to write $1 - \sin^2 x = \cos^2 x$. Then,

$$\frac{\cos^2 x}{\sin x \cos x} = \frac{\cos x}{\sin x} = \cot x.$$

15. Compute $\cot\left(\frac{2\pi}{3}\right)$.

Solution: For $\frac{2\pi}{3}$, $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$ and $\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$. Thus,

$$\cot\left(\frac{2\pi}{3}\right) = \frac{\cos\left(\frac{2\pi}{3}\right)}{\sin\left(\frac{2\pi}{3}\right)} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}.$$

16. Find the area of a sector with a central angle of $\frac{\pi}{3}$ radians in a circle with a radius of 10 cm.

Solution: The area of a sector is

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(10^2)\left(\frac{\pi}{3}\right) = \frac{100\pi}{6} = \frac{50\pi}{3} \text{ cm}^2.$$

17. Find the area of a sector with a central angle of 50° in a circle with a radius of 8 cm.

Solution: Convert 50° to radians:

$$50^\circ \times \frac{\pi}{180} = \frac{5\pi}{18}.$$

Then,

$$A = \frac{1}{2}(8^2)\left(\frac{5\pi}{18}\right) = \frac{32 \cdot 5\pi}{18} = \frac{160\pi}{18} = \frac{80\pi}{9}\ \text{cm}^2.$$

18. In a right triangle, if $\cos(\theta) = \frac{5}{13}$ for an acute angle θ, find $\sin(\theta)$.

Solution: Use the Pythagorean theorem:

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{5}{13}\right)^2} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}.$$

19. Suppose $\cot(\theta) = -\frac{5}{3}$ and that θ is in the fourth quadrant. Find the exact values of $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, $\csc(\theta)$, and $\sec(\theta)$.

Solution: Since θ is in quadrant IV, cosine is positive and sine is negative. Then,

$$\cos\theta = \frac{5}{\sqrt{34}}, \quad \sin\theta = -\frac{3}{\sqrt{34}}, \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{3}{5},$$

$$\csc\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{34}}{3}, \quad \sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{34}}{5}.$$