MATH 1150: Midterm 1 Practice (Solutions)

1. Simplify the expression completely: $\dfrac{2x}{x^2-4} - \dfrac{1}{x-2}$.

   Solution. Notice that
   $$x^2 - 4 = (x-2)(x+2).$$
   Rewrite the first fraction using the factorization:
   $$\frac{2x}{(x-2)(x+2)} - \frac{1}{x-2}.$$
   Express the second term with the common denominator $(x-2)(x+2)$:
   $$\frac{2x}{(x-2)(x+2)} - \frac{x+2}{(x-2)(x+2)} = \frac{2x-(x+2)}{(x-2)(x+2)}.$$
   Simplify the numerator:
   $$2x - (x+2) = x - 2.$$
   Then,
   $$\frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2},$$
   provided $x \neq 2$ (to avoid division by zero).

   □

2. Simplify the expression: $\left(\dfrac{4x^{-2}y^3}{2x^3y^{-1}}\right)^{-2}$.

   Solution. First simplify the expression inside the parentheses:
   $$\frac{4}{2} \cdot \frac{x^{-2}}{x^3} \cdot \frac{y^3}{y^{-1}} = 2 \cdot x^{-5} \cdot y^4.$$
   Now, raise to the power of -2:
   $$\left(2x^{-5}y^4\right)^{-2} = 2^{-2} \cdot x^{10} \cdot y^{-8} = \frac{x^{10}}{4y^8}.$$

   □

3. Solve for x if $|3-4x|=7$.

Solution. The absolute value equation splits into two cases.

Case 1: $3-4x=7$ Solve:

$$-4x=4 \quad \Longrightarrow \quad x=-1.$$

Case 2: $3-4x=-7$ Solve:

$$-4x=-10 \quad \Longrightarrow \quad x=\frac{5}{2}.$$

Thus, the solutions are $x=-1$ and $x=\frac{5}{2}$. □

4. Solve the inequality: $3|x+2|-5 \leq 10$.

Solution. First, add 5 to both sides:

$$3|x+2| \leq 15.$$

Divide by 3:

$$|x+2| \leq 5.$$

This inequality means:

$$-5 \leq x+2 \leq 5.$$

Subtract 2 from each part:

$$-7 \leq x \leq 3.$$

Thus, the solution set is $x \in [-7, 3]$.

□

5. Let

$$f(x) = \sqrt{2x - 4} + \frac{1}{x - 3}.$$

(a) Find the domain of f.

(b) Evaluate $f(5)$.

Solution.

(a) Domain:

- For the square root, require $2x - 4 \geq 0$, so $x \geq 2$.
- For the rational term, $x - 3 \neq 0$, so $x \neq 3$.

Thus, the domain is:

$$[2, 3) \cup (3, \infty).$$

(b) Evaluation: Plug in $x = 5$:

$$f(5) = \sqrt{2(5) - 4} + \frac{1}{5 - 3} = \sqrt{10 - 4} + \frac{1}{2} = \sqrt{6} + \frac{1}{2}.$$

$\square$

6. Consider the function

$$g(x) = \begin{cases} 2x - 1, & x < 1, \\ x^2, & x \geq 1. \end{cases}$$

(a) Evaluate $g(0)$, $g(1)$, and $g(-2)$.

(b) Solve for all x such that $g(x) = 4$.

Solution.

(a) Evaluation:

- For $x = 0$: Since $0 < 1$, use $2x - 1$. Thus, $g(0) = 2(0) - 1 = -1$.
- For $x = 1$: Since $1 \geq 1$, use x^2. Thus, $g(1) = 1^2 = 1$.
- For $x = -2$: Since $-2 < 1$, use $2x - 1$. Thus, $g(-2) = 2(-2) - 1 = -5$.

(b) Solving $g(x) = 4$: *Case 1:* $x < 1$ with $2x - 1 = 4$:

$$2x = 5 \implies x = \frac{5}{2},$$

but $\frac{5}{2} > 1$ (so discard this solution).
Case 2: $x \geq 1$ with $x^2 = 4$:

$$x = 2 \quad \text{or} \quad x = -2.$$

Since $x \geq 1$, only $x = 2$ is acceptable.
Thus, the only solution is $x = 2$.

$\square$

7. A function $h(t) = 15 + 4t$ gives the height (in centimeters) of a plant t days after planting. Explain the meaning of $h(5) = 35$.

Solution. The value $h(5) = 35$ means that 5 days after planting, the plant is 35 centimeters tall. $\square$

8. For the function $f(x) = x^2 - 3x + 2$, compute the difference quotient $\frac{f(x+h)-f(x)}{h}$, and simplify your answer.

Solution. First, compute $f(x+h)$:

$$\begin{aligned} f(x+h) &= (x+h)^2 - 3(x+h) + 2 \\ &= x^2 + 2xh + h^2 - 3x - 3h + 2. \end{aligned}$$

Then,

$$f(x+h) - f(x) = [x^2 + 2xh + h^2 - 3x - 3h + 2] - [x^2 - 3x + 2] = 2xh + h^2 - 3h.$$

Factor out h:

$$f(x+h) - f(x) = h(2x + h - 3).$$

Thus, the difference quotient is:

$$\frac{f(x+h) - f(x)}{h} = 2x + h - 3 \quad (\text{for } h \neq 0).$$

$\square$

9. Let $f(x) = 3x - 2 \quad$ and $\quad g(x) = x^2 + 1$. Find:

   (a) $(f + g)(x)$,

   (b) $(f \cdot g)(x)$, and

   (c) $\left(\frac{f}{g}\right)(x)$.

Solution.

(a)

$$(f + g)(x) = (3x - 2) + (x^2 + 1) = x^2 + 3x - 1.$$

(b)

$$(f \cdot g)(x) = (3x - 2)(x^2 + 1) = 3x^3 + 3x - 2x^2 - 2.$$

(c)

$$\left(\frac{f}{g}\right)(x) = \frac{3x - 2}{x^2 + 1}.$$

$\square$

10. Let $f(x) = \sqrt{x+4}$ and $g(x) = 2x - 1$.

(a) Find the formula for $(f \circ g)(x)$.

(b) Evaluate $(f \circ g)(3)$.

Solution.

(a)

$$(f \circ g)(x) = f(g(x)) = \sqrt{(2x-1)+4} = \sqrt{2x+3}.$$

(b) For $x = 3$:

$$(f \circ g)(3) = \sqrt{2(3)+3} = \sqrt{6+3} = \sqrt{9} = 3.$$

□

11. Consider the function $f(x) = |x|$. A new function $g(x)$ is obtained by performing the following transformations in order:

1. Shift $f(x)$ 3 units to the right,
2. Reflect it across the x-axis,
3. Vertically stretch it by a factor of 2,
4. Shift it upward by 5 units.

Write the formula for $g(x)$.

Solution. Apply each transformation step by step:

1. Shifting 3 units right gives: $|x - 3|$.
2. Reflecting across the x-axis: $-|x - 3|$.
3. Vertical stretch by 2: $-2|x - 3|$.
4. Shifting upward by 5: $-2|x - 3| + 5$.

Thus, the function is:

$$g(x) = -2|x - 3| + 5.$$

□

12. Determine whether the function $f(x) = x^3 - x$ is even, odd, or neither.

Solution. Test the definition by computing $f(-x)$:

$$f(-x) = (-x)^3 - (-x) = -x^3 + x = -(x^3 - x) = -f(x).$$

Since $f(-x) = -f(x)$ for all x, the function is **odd**.

□