Quadratic Functions

Introduction

Quadratic functions are fundamental mathematical models used in a variety of real-world contexts, such as physics, economics, and engineering. In this lecture, we will explore their properties, forms, and graphical representations.

Overview of Quadratic Function Forms

Form	Equation	Notes
General Form		a, b, and c are constants. Useful for identifying the y-intercept, c.
Vertex (Standard) Form		a determines the direction and width of the parabola. (h, k) is the vertex.
Factored Form		r_1 and r_2 are the roots (zeros) of the quadratic function. Useful for solving equations.

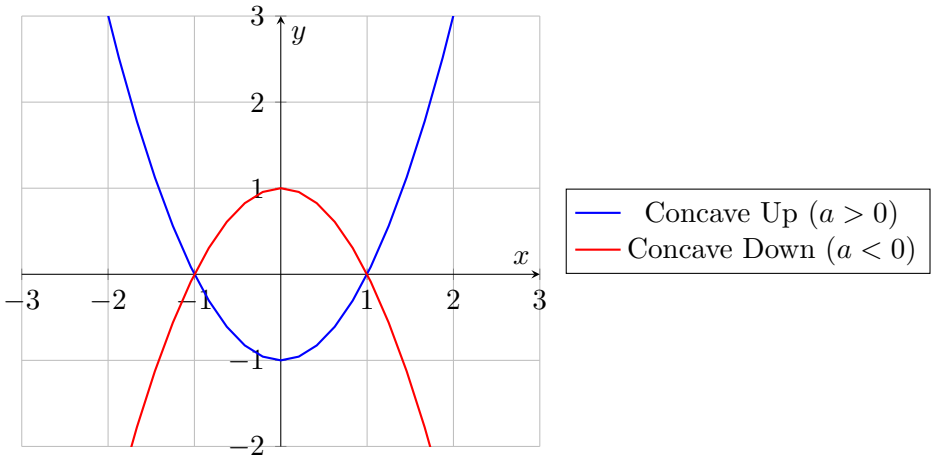
Remark. In general form,

- a _____ .
- b _____ .
- c _____ .

Concavity

The concavity of a quadratic function describes the direction the parabola opens.

- $a > 0$: _____ .
- $a < 0$: _____ .



Vertex and Maximum/Minimum

The vertex represents _____ .

- If $a > 0$, _____ .
- If $a < 0$, _____ .

Question. In standard form $y = a(x - h)^2 + k$, the vertex is (h, k). How can we find the vertex if the function is in general form $y = ax^2 + bx + c$?

Finding the Vertex

Example. Find the vertex of $y = 2x^2 - 4x + 1$.

Converting General Form to Standard Form

Example. Convert $y = 2x^2 - 4x + 1$ to standard form.

Finding Intercepts

Example. Find the x-intercepts and y-intercept of the quadratic function: $f(x) = 2x^2 - 4x + 1$.

Example. Find a function f whose graph is a parabola with the given vertex and that passes through the given point.

- Vertex: $(2, -3)$
- Point: $(4, 5)$

Graphing Quadratic Functions

Example. Graph the quadratic function $f(x) = x^2 - 2x - 3$ and determine its domain and range.

Application: Projectile Motion

In physics, the motion of an object under the influence of gravity, such as a ball thrown in the air, can be modeled by a quadratic function. The equation for the height $h(t)$ of the object at time t is typically given by:

$$h(t) = -\frac{1}{2}gt^2 + v_0t + h_0,$$

where:

- g is the acceleration due to gravity (approximately 9.8 m/s^2 on Earth).
- v_0 is the initial velocity of the object (in m/s).
- h_0 is the initial height of the object (in m).

Example. A ball is thrown upward with an initial velocity of 20 m/s from a height of 2 m. The height of the ball at any time t seconds is given by:

$$h(t) = -4.9t^2 + 20t + 2.$$

Find:

1. The maximum height of the ball.
2. The time it takes for the ball to hit the ground.