Introduction

Linear functions are some of the simplest and most widely used functions in mathematics. They are used to model real-world relationships, such as distance over time or cost over quantity. This lecture covers the fundamentals of linear functions, how to find their equations, and how to apply them to solve problems.

Overview of Line Forms

Form	**Equation**	**Notes**
Slope-Intercept	$y = mx + b$	m is the slope, b is the y-intercept
Point-Slope	$y - y_1 = m(x - x_1)$	m is the slope, (x_1, y_1) is a point on the line
General	$ax + by = c$	a, b, c are constants
Horizontal Line	$y = c$	$m = 0$, constant y-value
Vertical Line	$x = c$	Undefined slope, constant x-value

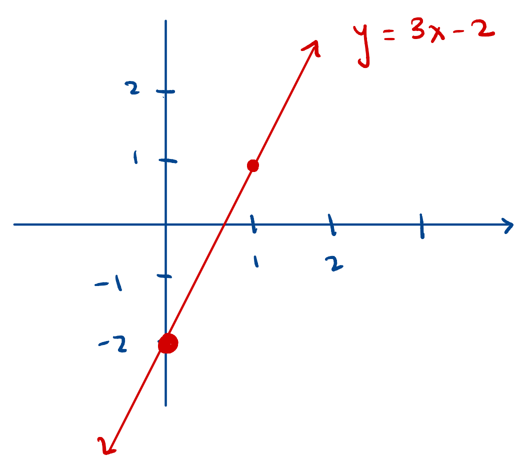
Slope-Intercept Form

Example. Write the equation of a line with a slope of 3 and a y-intercept of -2.

Here, $m = 3$ and $b = -2$

Slope-intercept form: $y = mx + b$

$y = 3x - 2$

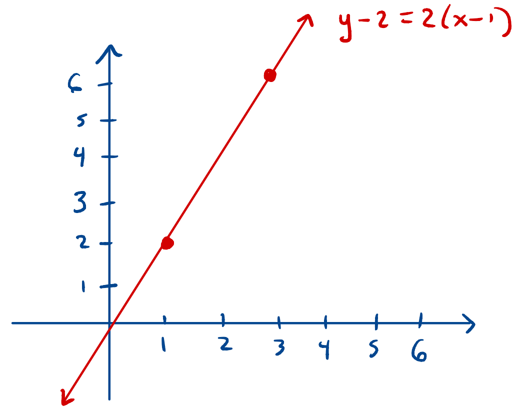


Point-Slope Form

Example. Find the equation of the line passing through the points $(1, 2)$ and $(3, 6)$.

① Calculate the slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6-2}{3-1} = \dfrac{4}{2} = 2$

② Use point-slope form with $(x_1, y_1) = (1, 2)$



$$y - y_1 = m(x - x_1)$$

$$y - 2 = 2(x - 1)$$

Two ways to write the same line

③ What is the slope-intercept form?

$$y - 2 = 2x - 2$$

$$y = 2x$$

Parallel and Perpendicular Lines

Definition.

- Two lines are parallel if they have the same slope.
- Two lines are perpendicular if the product of their slopes is -1.

Example: Determine if the lines $y = 2x + 3$ and $y = -\frac{1}{2}x - 4$ are perpendicular.

$m_1 = 2$ and $m_2 = -\frac{1}{2}$

$m_1 \cdot m_2 = 2 \cdot \left(-\frac{1}{2}\right) = -1$

$\boxed{\text{Perpendicular.}}$

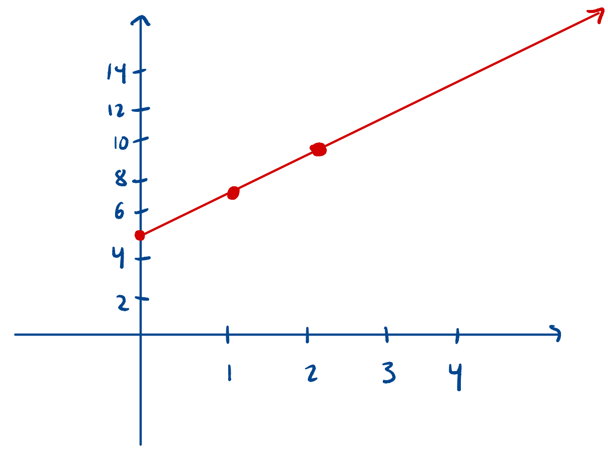
Modeling Situations with Linear Functions

Example: A company charges a flat fee of \$5 for delivery and \$2 per mile. Write a linear function that models the total cost C based on the number of miles x.

Flat fee = \$5 ← y-intercept

Rate per mile = \$2 ← slope

Function: $C(x) = 2x + 5$



General Form of a Line

Example. Convert $y = 2x + 3$ to general form. → $ax + by = c$

$y = 2x + 3$

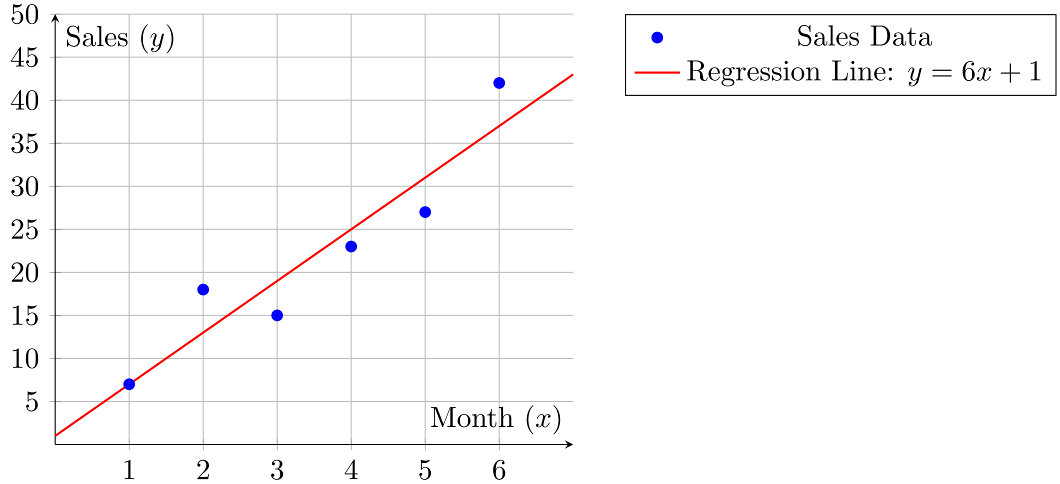
$-2x + y = 3$ OR $2x - y = -3$ (either one works)

Application: Data Analysis & Trend Lines

Linear models are essential in data analysis for understanding trends and making predictions. Imagine a company tracks its sales over six months. The data points are:

Month (x)	Sales (y) in \$1000s
1	7
2	18
3	15
4	23
5	27
6	42

The scatter plot below shows this data, and the regression line models the trend.



The regression line is $y = 6x + 1$. Using the regression line, we can predict future sales. For example, what does the model predict for the sales in Month 8?

$$y = 6x + 1$$

$$y = 6 \cdot (8) + 1$$

$$y = 48 + 1$$

$$y = 49$$

$\boxed{\$49,000}$