Even and Odd Functions

Introduction

Functions are classified as even, odd, or neither based on their symmetry properties. Understanding these classifications is essential for analyzing graphs and solving equations.

Definitions of Even and Odd Functions

Type of Function	Definition	Geometric Interpretation
Even	$f(-x) = f(x)$ for all x in the domain of f	Symmetric about the y-axis.
Odd	$f(-x) = -f(x)$ for all x in the domain of f	180° symmetry about the origin
Neither	$f(x)$ satisfies neither condition.	No symmetry

Examples of Even and Odd Functions

Example. $f(x) = x^2$

$$f(-x) = (-x)^2 = x^2 = f(x)$$ Even

Example. $g(x) = x^3$

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$ Odd

Example. $h(x) = x^3 + x^2$

$$h(-x) = (-x)^3 + (-x)^2 = -x^3 + x^2$$ Neither

$f(x) = -x^2$

$f(-x) = -(-x)^2$

$= -x^2$

$\boxed{\text{Even}}$

Example. $f(x) = \dfrac{1}{x^2}$

$$f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x) \quad \underline{\text{Even}}$$

$$f(x) = \frac{x^3}{x^5 + x}$$

$$f(-x) = \frac{(-x)^3}{(-x)^5 + (-x)} = \frac{-x^3}{-x^5 - x}$$

$$= \frac{-1}{-1} \cdot \frac{x^3}{x^5 + x}$$

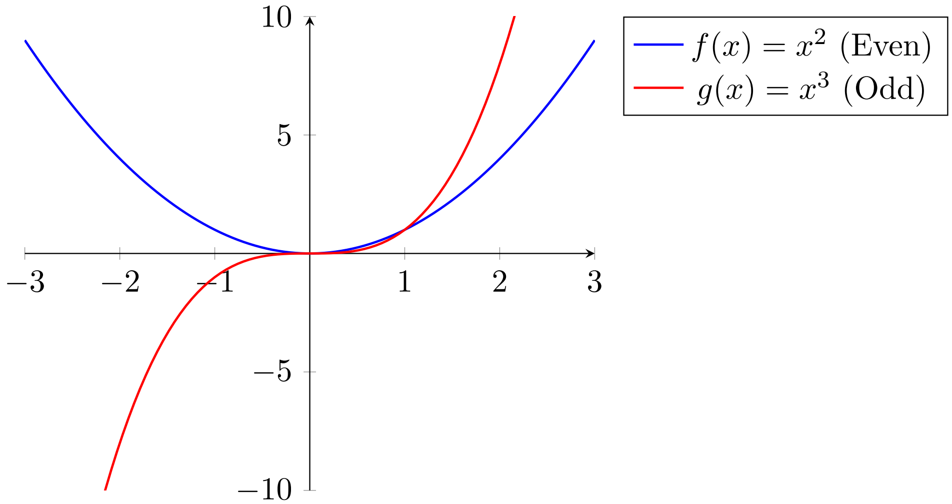
$$\underline{\text{Even}} \qquad = f(x)$$

Example. $g(x) = \dfrac{x}{x^2 + 1}$

$$g(-x) = \frac{-x}{(-x)^2 + 1} = \frac{-x}{x^2 + 1} = -\left(\frac{x}{x^2+1}\right) = -g(x) \quad \underline{\text{odd}}$$

Graphical Interpretation of Even and Odd Functions

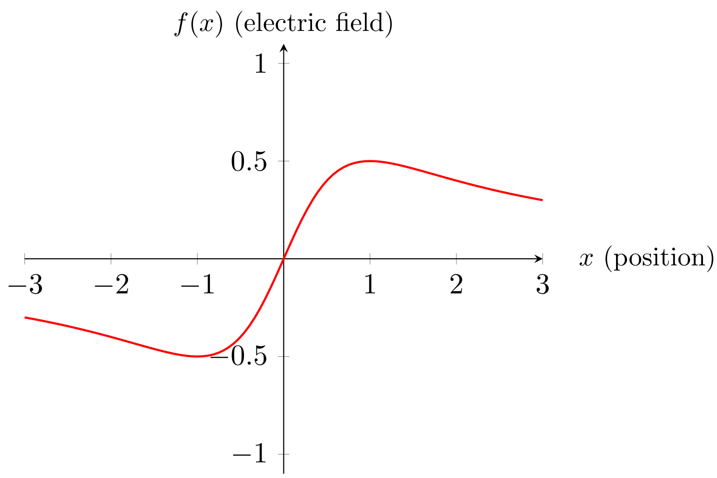
- Even functions are <u>symmetric about the y-axis</u>.
- Odd functions are <u>180° symmetry about the origin</u>.



Application: Electric Field of a Charged Particle

An electric field points away from a positive charge and toward a negative charge.

We can model this by the function $f(x) = \dfrac{x}{x^2+1}$.



- On the right side of the charge, the field is positive (points to the right).
- On the left side of the charge, the field is negative (points to the left).
- At $x = 0$, the electric field is zero.

For the electric field:

$$f(-x) = \frac{-x}{(-x)^2+1} = -\frac{x}{x^2+1} = -f(x).$$

This symmetry means the electric field has equal magnitude but opposite direction on either side of the charge.