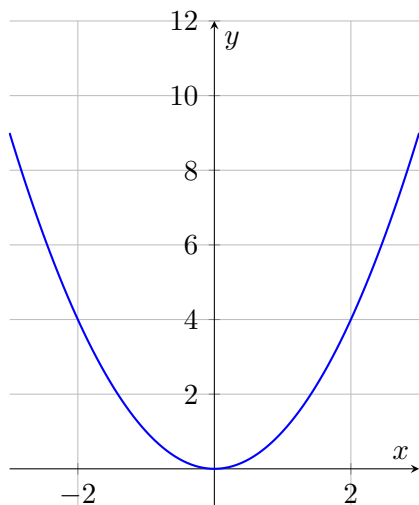# Transformations of Functions

## Introduction

Transformations allow us to modify functions to shift, stretch, compress, or reflect their graphs. Understanding transformations is key to graphing functions quickly and interpreting their behavior.

## Types of Transformations

**1. Vertical Transformations:**

| Transformation | Formula | What to do to the Coordinates |
|---|---|---|
| Vertical shift (up $c$) | | |
| Vertical shift (down $c$) | | |
| Vertical stretch | | |
| Vertical shrink | | |
| Reflection about the $x$-axis | | |

**Example.** Consider the quadratic function $f(x) = x^2$. A **vertical shift up by 3** transforms the function to $f(x) + 3 = x^2 + 3$.
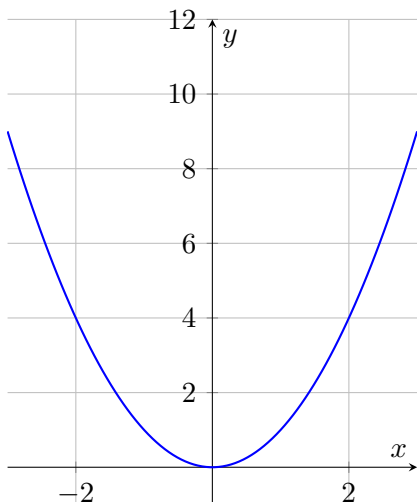


| $x$ | $f(x) = x^2$ | $f(x) + 3$ |
|---|---|---|
| $-3$ | 9 | |
| $-2$ | 4 | |
| $-1$ | 1 | |
| 0 | 0 | |
| 1 | 1 | |
| 2 | 4 | |
| 3 | 9 | |

**2. Horizontal Transformations:**

| Transformation | Formula | What to do to the Coordinates |
|---|---|---|
| Horizontal shift (right $c$) | | |
| Horizontal shift (left $c$) | | |
| Horizontal stretch | | |
| Horizontal shrink | | |
| Reflection about the $y$-axis | | |

**Example.** Consider the quadratic function $f(x) = x^2$. A **horizontal shrink by a factor of 2** transforms the function to $f(2x)$.
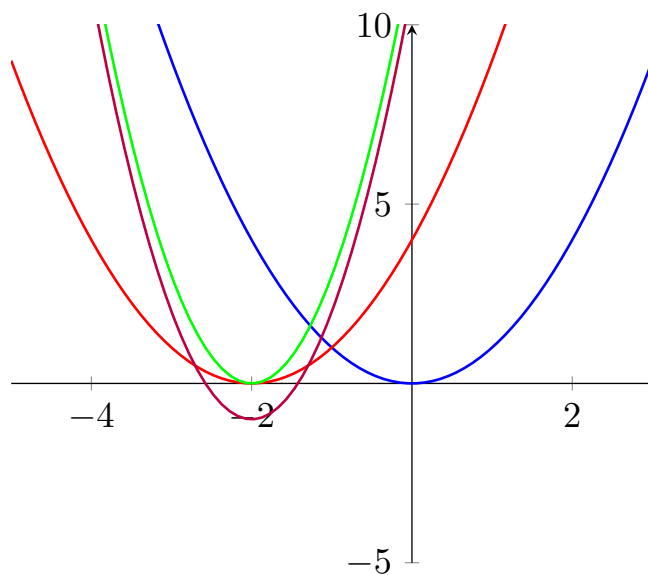


| $x$ | $f(x) = x^2$ | $f(2x)$ |
|---|---|---|
| $-3$ | 9 | |
| $-2$ | 4 | |
| $-1$ | 1 | |
| 0 | 0 | |
| 1 | 1 | |
| 2 | 4 | |
| 3 | 9 | |

## Sequences of Transformations

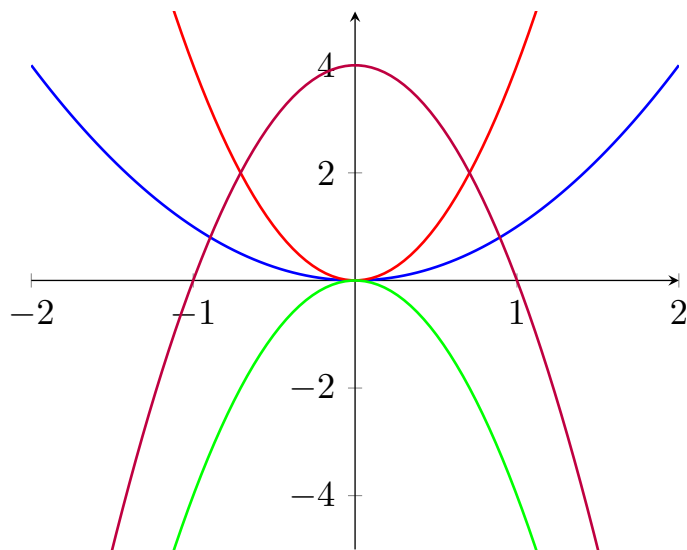**Question.** How do we apply more than one transformation at a time?

**Example:** Let $f(x) = x^2$. Apply the following transformations in order:

1. Horizontal shift left 2.

2. Vertical stretch by 3.

3. Vertical shift down 1.

**Example.** Let $f(x) = x^2$. Apply the following transformations in order:

1. Horizontal shrink by a factor of 2.

2. Reflection about the $x$-axis.

3. Vertical shift up 4.

**Example.** Let $f(x) = x^2$. Apply the following transformations in order:

1. Vertical shift up by 2.

2. Horizontal stretch by a factor of 2.

3. Vertical stretch by a factor of 3.

What is a simplified formula for the resulting transformation?
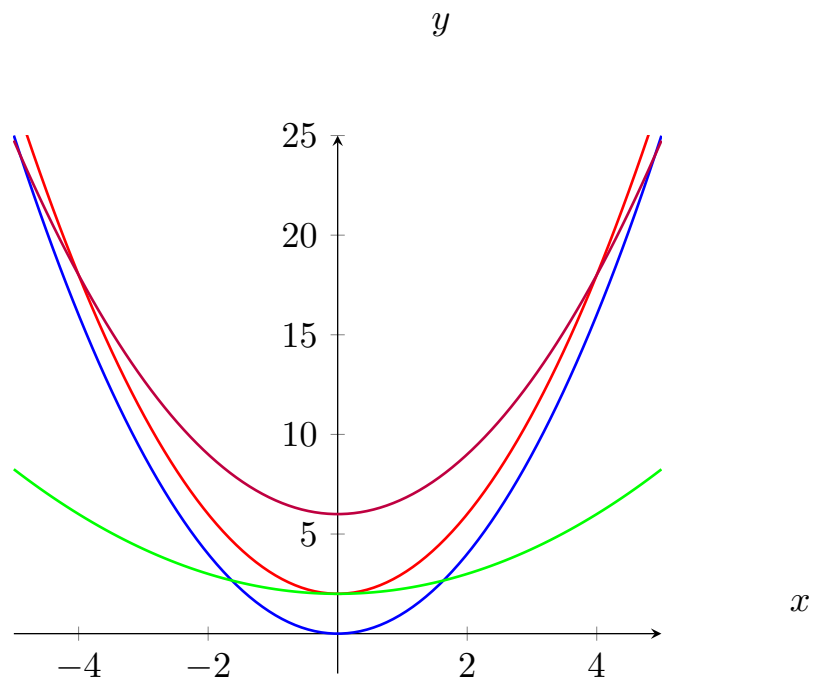
**Example.** Given the transformed function:

$$g(x) = -\sqrt{x - 3} + 2$$

Identify a parent function $f(x)$ and state the transformations, in order, needed to get from $f(x)$ to $g(x)$.

**Example.** Given the transformed function:

$$h(x) = 2(x + 1)^2 - 4$$

Identify a parent function $f(x)$ and state the transformations, in order, needed to get from $f(x)$ to $h(x)$.