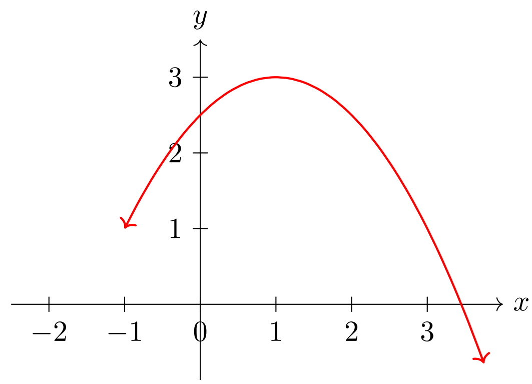
Limits

Definition. A **limit** describes the value a function approaches as the input approaches a specific value. In notation, we write

$$\lim_{x \to a} f(x) = L$$

to mean that as x gets closer and closer to a, the values of $f(x)$ get closer and closer to L.

Example. Compute $\lim_{x \to 1} -\frac{1}{2}(x-1)^2 + 3$



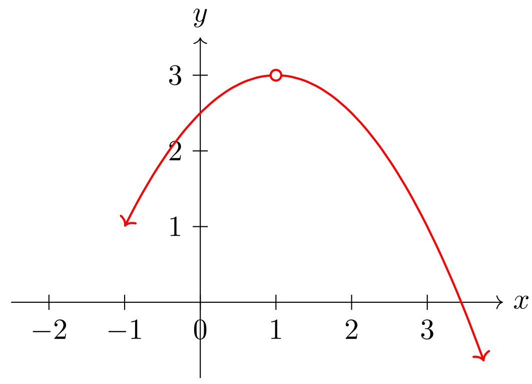
Definition. One-Sided Limits:

- The **left-hand limit** $\lim_{x \to a^-} f(x)$ is the value the function approaches as x approaches a from the left (that is, from values smaller than a).
- The **right-hand limit** $\lim_{x \to a^+} f(x)$ is the value the function approaches as x approaches a from the right (that is, from values larger than a).

Two-Sided Limits:

- The **two-sided limit** $\lim_{x \to a} f(x)$ exists if and only if both the left-hand and right-hand limits exist and are equal.

Example. Compute $\lim_{x \to 1} f(x)$



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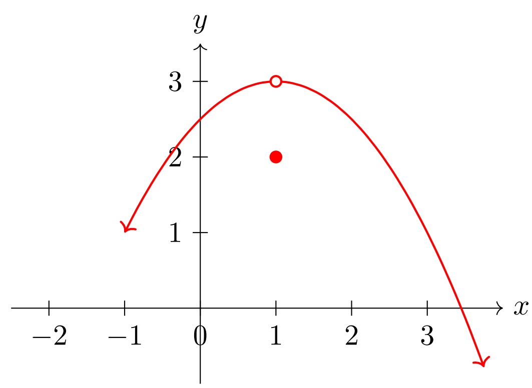

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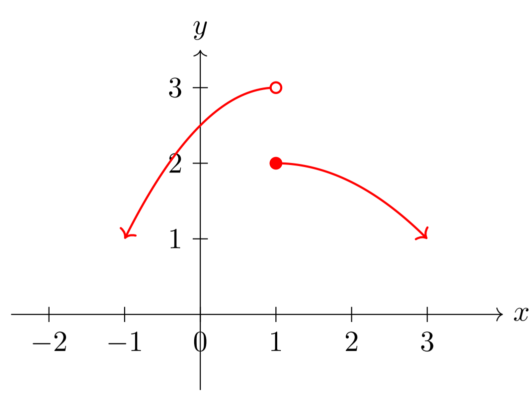
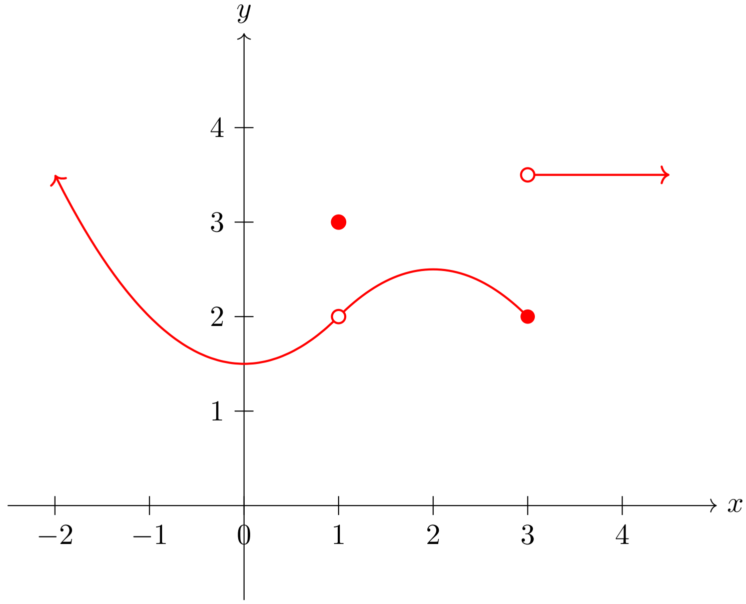


Example. The graph of a function $f(x)$ is shown below.



Answer the following:

(a) $\lim_{x \to -1^-} f(x) =$ ________

(b) $\lim_{x \to -1^+} f(x) =$ ________

(c) $\lim_{x \to -1} f(x) =$ ________

(d) $f(-1) =$ ________

(e) $\lim_{x \to 1^-} f(x) =$ ________

(f) $\lim_{x \to 1^+} f(x) =$ ________

(g) $\lim_{x \to 1} f(x) =$ ________

(h) $f(1) =$ ________

(i) $\lim_{x \to 3^-} f(x) =$ ________

(j) $\lim_{x \to 3^+} f(x) =$ ________

(k) $\lim_{x \to 3} f(x) =$ ________

(l) $f(3) =$ ________

Example. Evaluate the limit by factoring:

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}.$$

Example. Evaluate the limit by multiplying by the conjugate:

$$\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}.$$