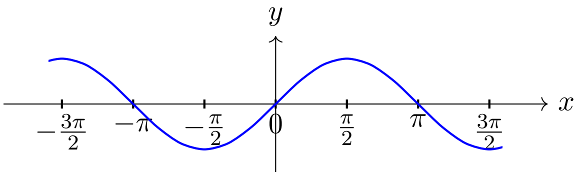
Trigonometric Graphs (again)

Definition. The **period** of a trigonometric function is the horizontal length it takes for the function to complete one full cycle and begin repeating its shape.

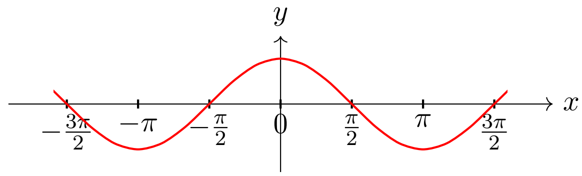
Definition. The **amplitude** of a trigonometric function is the distance from the middle of the graph (the midline) to the highest or lowest point. It tells you how tall the waves of the graph are.

Sine Function



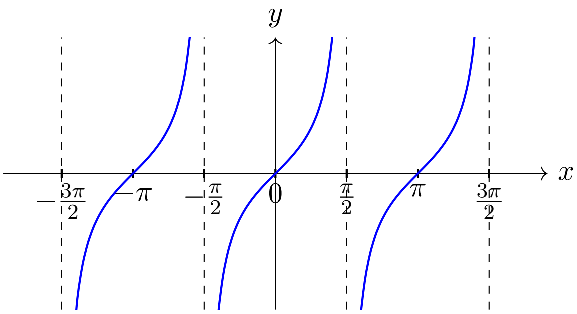
Amplitude: 1 Period: 2π

Cosine Function



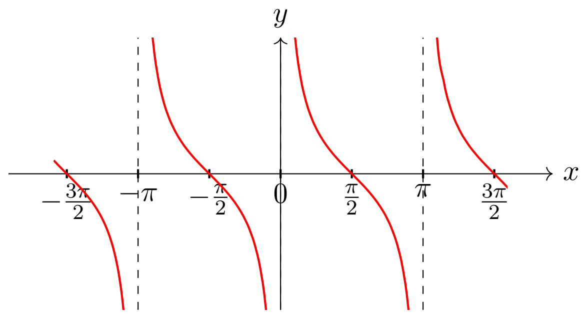
Amplitude: 1 Period: 2π

Tangent Function



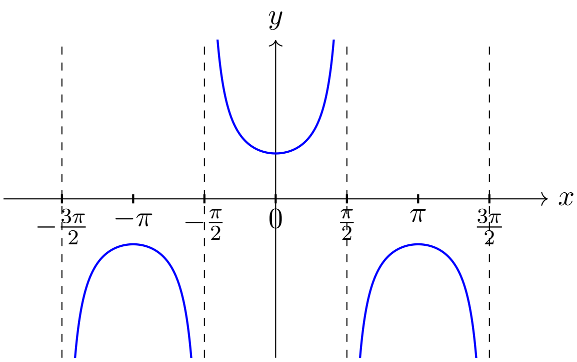
Amplitude: undefined Period: π

Cotangent Function



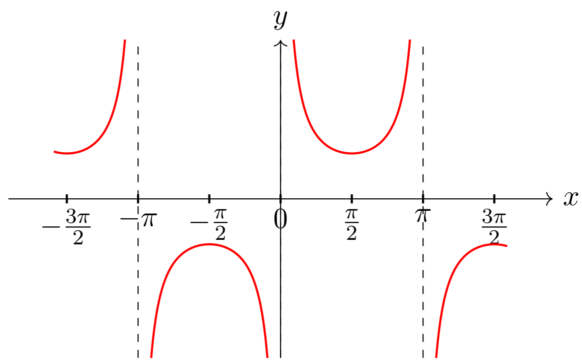
Amplitude: undefined Period: π

Secant Function



Amplitude: undefined Period: 2π

Cosecant Function



Amplitude: undefined Period: 2π

Definition. A **transformation** of the sine or cosine function has the general form

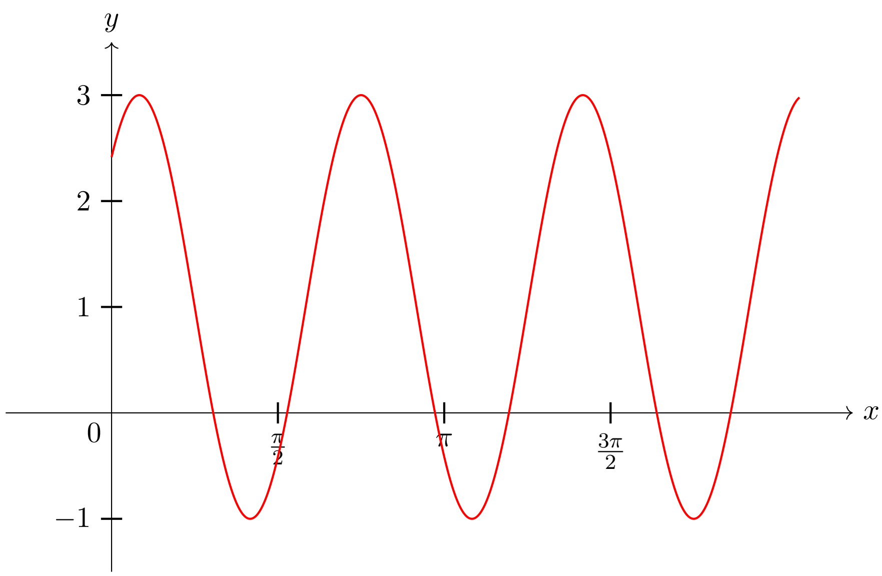
$$y = a\,\sin\big(b(x-c)\big) + d \quad \text{or} \quad y = a\,\cos\big(b(x-c)\big) + d,$$

Each letter controls a different part of the graph:

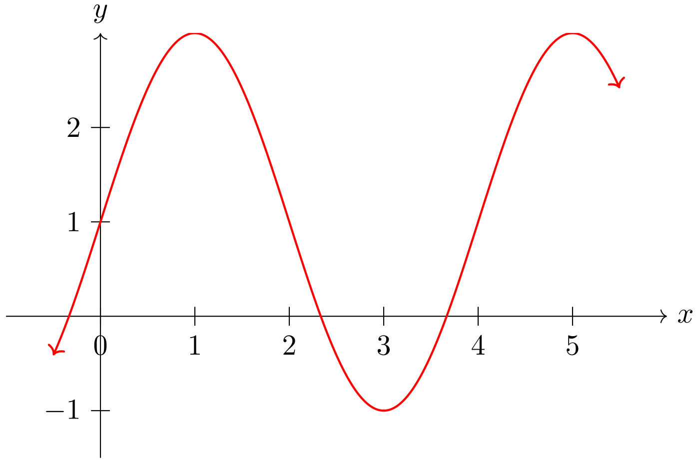
- a changes the **amplitude**, which affects how tall the waves are.
- b changes the **period**, or how wide each wave is. The period becomes $\dfrac{2\pi}{|b|}$.
- c causes a **horizontal shift** (called a phase shift). If $c > 0$, the graph shifts right. If $c < 0$, it shifts left.
- d causes a **vertical shift**, moving the whole graph up or down.

If $a < 0$, the graph is also **reflected across the x-axis**.

Example. Find the period, amplitude, and phase shift of the function $y = -2\sin\left(3\left(x - \frac{\pi}{4}\right)\right) + 1$.



Example. Find an equation for the following transformation of $\cos(x)$.



Example. Find the period of each function below. Write your answer in the blank space provided.

1. $y = 3\cot(3\pi x)$ ________________

2. $y = 3\csc\left(\frac{3x}{2}\right)$ ________________

3. $y = -5\tan\left(\frac{x}{2} - \frac{\pi}{3}\right)$ ________________