FCQs: If our section gets a 90% response rate, everyone gets a free 20/20 in the FCQ extra credit quiz column.

Angle Sum & Difference Identities

Definition (Sum & Difference Identities)**.**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

Remark. You do not need to memorize the half angle identities. These identities will be listed on a provided formula sheet for the exam. You are responsible for memorizing the reciprocal, quotient, and Pythagorean identities.

Example. Evaluate $\cos(75°)$.

Write $75° = 45° + 30°$

$$\cos(75°) = \cos(45° + 30°) = \cos(45°)\cos(30°) - \sin(45°)\sin(30°)$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

Example. Evaluate $\sin\left(\frac{11\pi}{12}\right)$.

$$\frac{11\pi}{12} = \frac{2\pi}{3} + \frac{\pi}{4}$$

A B

$$\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) = \sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$
$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right) \cdot \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

Example. Given $\sin A = \frac{3}{5}$, where A is in Quadrant I, and $\cos B = -\frac{5}{13}$, where B is in Quadrant II, find $\sin(A+B)$, $\cos(A+B)$, and determine the quadrant of $A+B$.

Determine $\cos A$ and $\sin B$ (using $\cos^2\theta + \sin^2\theta = 1$):

$$\cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\left(\frac{3}{5}\right)^2} = \frac{4}{5} \quad \text{(positive since in Quadrant I)}$$

$$\sin B = \sqrt{1-\cos^2 B} = \sqrt{1-\left(-\frac{5}{13}\right)^2} = \frac{12}{13} \quad \text{(positive since in Quadrant II)}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B = \frac{3}{5}\cdot\left(-\frac{5}{13}\right) + \frac{4}{5}\cdot\frac{12}{13} = \frac{-15+48}{65} = \frac{33}{65}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B = \frac{4}{5}\cdot\left(-\frac{5}{13}\right) - \frac{3}{5}\cdot\frac{12}{13} = \frac{-20-36}{65} = -\frac{56}{65}$$

Since $\sin(A+B) > 0$ and $\cos(A+B) < 0$, the angle $A+B$ is in Quadrant II.