

Half Angle Identities

Definition (Half Angle Identities).

$$\begin{aligned}\sin\left(\frac{\theta}{2}\right) &= \pm\sqrt{\frac{1-\cos\theta}{2}} \\ \cos\left(\frac{\theta}{2}\right) &= \pm\sqrt{\frac{1+\cos\theta}{2}} \\ \tan\left(\frac{\theta}{2}\right) &= \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}\end{aligned}$$

Remark. The sign of the square root depends on the quadrant in which $\frac{\theta}{2}$ lies.

Remark. You do not need to memorize the half angle identities. These identities will be listed on a provided formula sheet for the exam. You are responsible for memorizing the reciprocal, quotient, and Pythagorean identities.

Example. Given a right triangle where $\cos\theta = \frac{3}{5}$ and θ is in Quadrant I, evaluate $\sin\left(\frac{\theta}{2}\right)$.

Example. Evaluate $\cos(15^\circ)$ using a half angle identity.

Example. Evaluate $\sin\left(\frac{5\pi}{8}\right)$ using a half angle identity.

Example. Use a half-angle identity to solve the equation

$$\sin^2\left(\frac{x}{2}\right) = \frac{3}{4}, \quad 0 \leq x < 2\pi.$$