Double Angle Identities

Double angle identities allow us to express trigonometric functions of $2x$ in terms of functions of x. These identities are useful in simplifying expressions, solving equations, and evaluating trigonometric functions without a calculator.

Definition. The double angle identities for sine, cosine, and tangent are:

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)} \quad \text{provided } \tan(x) \neq \pm 1$$

Remark. You do not need to memorize the double angle identities. These identities will be listed on a provided formula sheet for the exam. You are responsible for memorizing the reciprocal, quotient, and Pythagorean identities.

Example. Suppose θ is an angle in a right triangle such that $\sin(\theta) = \frac{3}{5}$ and θ is in Quadrant I. Find $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$.

Example. Given that $\cos(x) = -\frac{5}{13}$ and x is in Quadrant II, find $\sin(2x)$, $\cos(2x)$, and $\tan(2x)$.

Example. Suppose $\sin(2x) > 0$ and $\cos(2x) < 0$. In which quadrant does the angle $2x$ lie?

Example. Solve the equation $\sin(x)\cos(x) = \frac{1}{4}$ for all solutions in the interval $[0, 2\pi)$.