

Functions

Introduction

Functions are one of the most powerful tools in mathematics, providing a way to model relationships and behaviors in the real world. This lecture will deepen your understanding of functions by exploring their definitions, domains, and ranges.

Definition of a Function

Definition. What is a function?

Example. Let $f(x) = 2x + 3$. Find $f(4)$.

Example. Evaluate $f(x) = x^2 - 5x + 6$ for $x = 3$.

Definition. A **piecewise function** is a function described by _____ ,

each applying to a specific interval of the domain:

$$f(x) = \begin{cases} f_1(x), & \text{if } x \text{ is in } I_1, \\ f_2(x), & \text{if } x \text{ is in } I_2, \\ \vdots & \\ f_n(x), & \text{if } x \text{ is in } I_n. \end{cases}$$

Example. Evaluate the piecewise function:

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 0, \\ -2x & \text{if } x < 0. \end{cases}$$

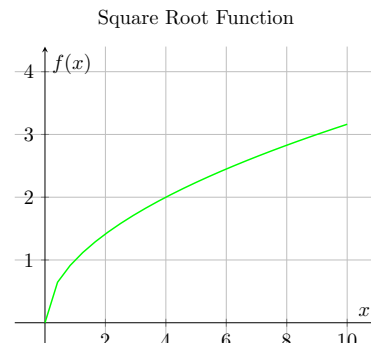
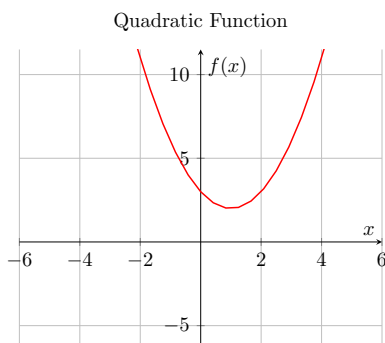
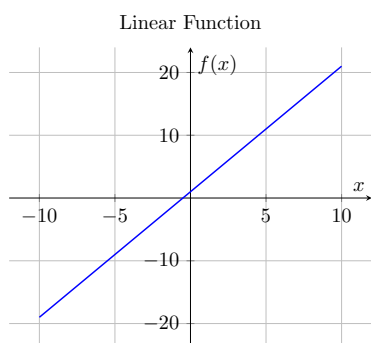
Domain and Range

Definition. The domain of a function $f(x)$ is _____
for which the function is defined.

Definition. The range of a function $f(x)$ is _____
the function can produce.

Example.

Function Type	Domain	Range
Linear Function: $f(x) = mx + b$		
Quadratic Function: $f(x) = ax^2 + bx + c$		
Square Root Function: $f(x) = \sqrt{x}$		



Example. Find the domain and range of $f(x) = \sqrt{x+2}$

Quadratic Functions and the Vertex Formula

Vertex Formula for Quadratic Functions
For $f(x) = ax^2 + bx + c$, the vertex is given by: $x = -\frac{b}{2a}, \quad f(x) = f\left(-\frac{b}{2a}\right).$

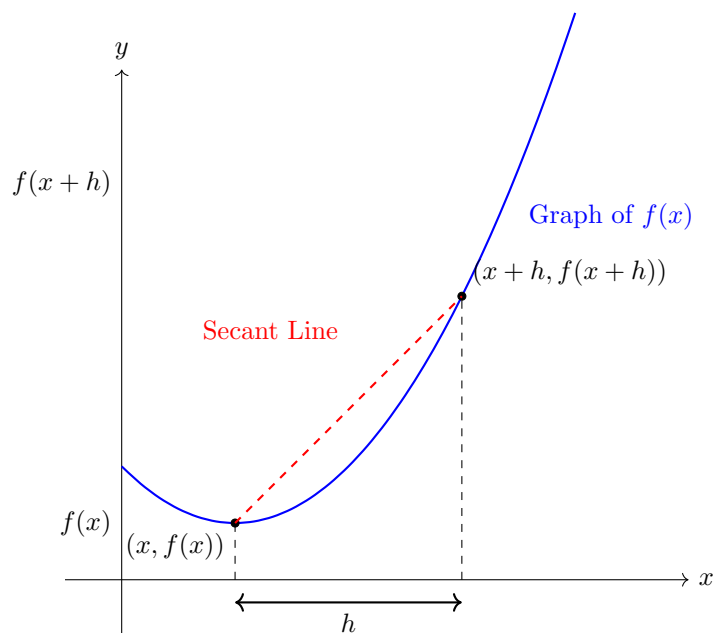
Example: Find the vertex of $f(x) = x^2 - 6x + 5$.

Example. Find the range of $f(x) = x^2 - 4$.

The Difference Quotient

Definition. The difference quotient is a measure of the average rate of change of a function over an interval. It represents the slope of the secant line between two points on the graph of a function $f(x)$ separated by a horizontal distance h :

$$\frac{f(x+h) - f(x)}{h}, \quad h \neq 0.$$



Example. Compute the difference quotient for $f(x) = x^2$

Example. Compute the difference quotient for $f(x) = 3x - 4$

Applications

Example. The drug dosage $D(w)$ that a doctor prescribes is a function of the patient's weight w .

Example. The amount of money earned $E(h)$ is a function of the number of hours h worked.