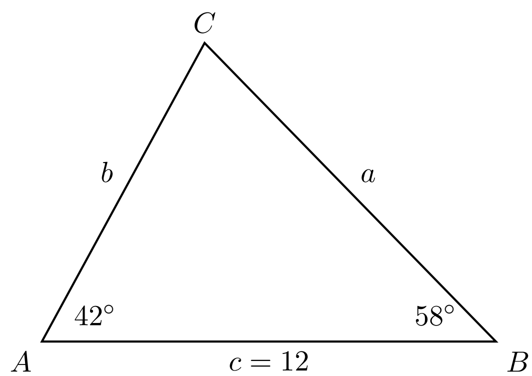
## Laws of Sines and Cosines

**Theorem** (Law of Sines)**.** In any triangle with angles $A$, $B$, and $C$, and opposite sides $a$, $b$, and $c$, respectively, the Law of Sines states:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

**Example.** Use the Law of Sines to find all sides and angles of the given triangle.



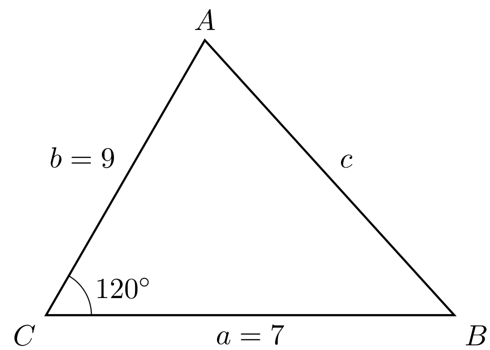
**Theorem** (Law of Cosines)**.** In any triangle with sides $a$, $b$, and $c$, and corresponding opposite angles $A$, $B$, and $C$, the Law of Cosines states:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

This can be rewritten for other sides:

$$a^2 = b^2 + c^2 - 2bc\cos A \quad \text{and} \quad b^2 = a^2 + c^2 - 2ac\cos B$$

**Example.** Use the Law of Cosines to find all sides and angles of the given triangle.



**Theorem** (Ambiguous Case for the Law of Sines)**.** When using the Law of Sines to solve for an angle, applying the inverse sine function may lead to *two possible angle values*, resulting in what is known as the **ambiguous case**.

**Possible outcomes:**

- **Two triangles:** If both $\theta$ and $180^\circ - \theta$ lead to valid triangle angle measures, then two distinct triangles are possible.
- **One triangle:** If only one of the two angle values leads to a valid triangle, then exactly one triangle is possible.
- **No triangle:** If $\sin\theta > 1$ or neither angle produces valid measures, then no triangle can be formed.

**Example.** Find all sides and angles of the given triangle.