

Inverse Trigonometric Identities

Definition. The inverse sine function $\sin^{-1}(x)$, also written as $\arcsin(x)$, is the angle θ such that $\sin(\theta) = x$, where θ is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

$$\text{Domain: } -1 \leq x \leq 1 \quad \text{Range: } -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

Definition. The inverse cosine function $\cos^{-1}(x)$, also written as $\arccos(x)$, is the angle θ such that $\cos(\theta) = x$, where θ is between 0 and π .

$$\text{Domain: } -1 \leq x \leq 1 \quad \text{Range: } 0 \leq \theta \leq \pi$$

Definition. The inverse tangent function $\tan^{-1}(x)$, also written as $\arctan(x)$, is the angle θ such that $\tan(\theta) = x$, where θ is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (but not equal to $\pm\frac{\pi}{2}$).

$$\text{Domain: all real numbers } x \in \mathbb{R} \quad \text{Range: } -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

Example. Evaluate $\cos^{-1}(\cos(\frac{\pi}{3}))$

Example. Evaluate $\sin^{-1}(\sin(\frac{5\pi}{4}))$

True/False: For any θ in $(-\infty, \infty)$, the following identity holds: $\sin^{-1}(\sin(\theta)) = \theta$.

True/False: For any x in $[-1, 1]$, the following identity holds: $\sin(\sin^{-1}(x)) = x$.

Example. Evaluate $\cos\left(\sin^{-1}\left(\frac{5}{13}\right)\right)$

Example. Rewrite the expression as an algebraic expression in terms of x :

$$\tan\left(\cos^{-1}(x)\right)$$