

Inverse Trigonometric Functions

The trig functions $\sin(\theta)$ and $\cos(\theta)$ take an angle and tell you a point on the unit circle:

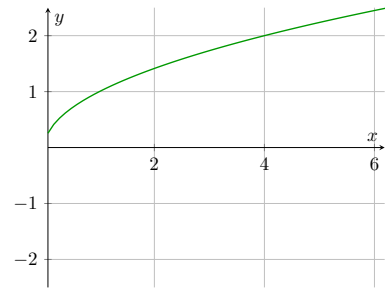
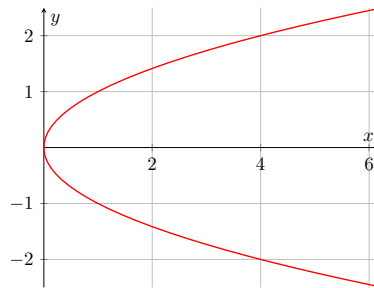
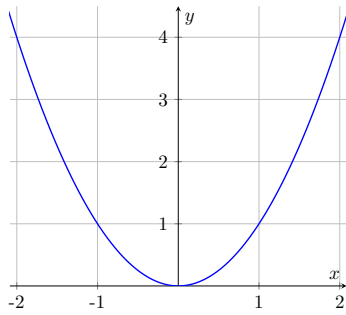
$\sin(\theta)$ is the y -coordinate of the point at angle θ

$\cos(\theta)$ is the x -coordinate of the point at angle θ

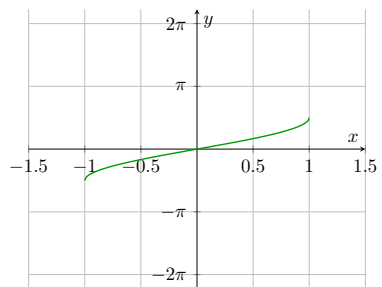
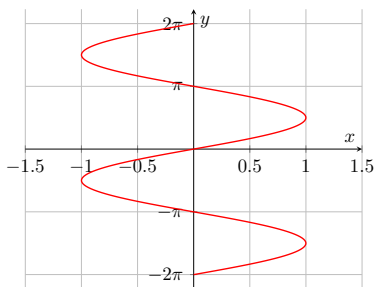
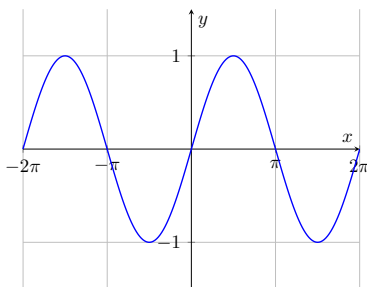
Question: Can we define functions to go the other way? Given an x - or y -coordinate on the unit circle, can we output the angle θ on the unit circle with that particular x - or y -coordinate?

Issue: There are infinitely many angles with a particular x - or y -coordinate. To define a function, we can only have one output.

How we answered this question with $f(x) = x^2$



The inverse of $\sin(x)$



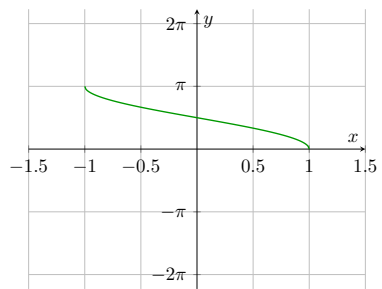
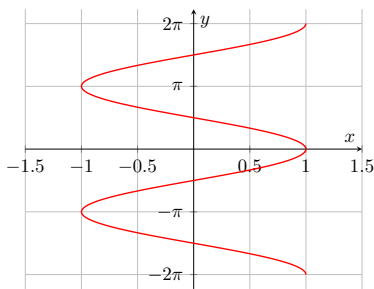
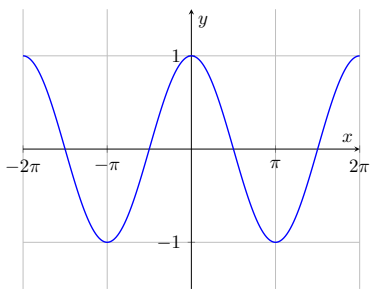
Definition. The **inverse sine function**, written as $\arcsin(x)$ or $\sin^{-1}(x)$, tells you the angle whose sine is x .

We only output angles between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, because that is an interval where the sine graph doesn't repeat.

Question. What is the domain of $\arcsin(x)$?

Question. What is the range of $\arcsin(x)$?

The inverse of $\cos(x)$



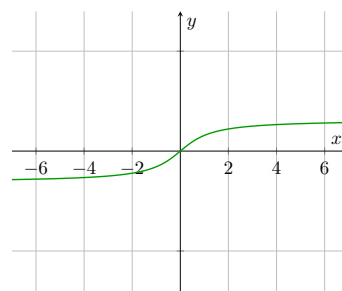
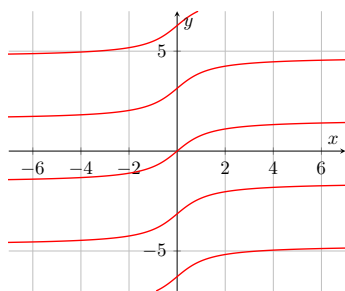
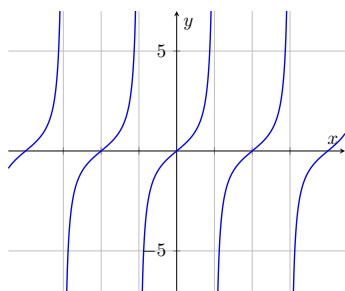
Definition. The **inverse cosine function**, written as $\arccos(x)$ or $\cos^{-1}(x)$, tells you the angle whose cosine is x .

We only output angles between 0 and π , because that is an interval where the cosine graph doesn't repeat.

Question. What is the domain of $\arccos(x)$?

Question. What is the range of $\arccos(x)$?

The inverse of $\tan(x)$



Definition. The **inverse tangent function**, written as $\arctan(x)$ or $\tan^{-1}(x)$, tells you the angle whose sine is x .

We only output angles between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, because that is an interval where the tangent graph doesn't repeat.

Question. What is the domain of $\arctan(x)$?

Question. What is the range of $\arctan(x)$?