Trigonometric Identities

This section covers fundamental trigonometric identities: the Pythagorean, reciprocal, quotient, even/odd, and cofunction identities.

Pythagorean Identities

Definition. The fundamental identity derived from the unit circle $x^2 + y^2 = 1$ is:

$$\sin^2\theta + \cos^2\theta = 1.$$

From this, we obtain two additional identities:

$$1 + \tan^2\theta = \sec^2\theta \quad \text{and} \quad 1 + \cot^2\theta = \csc^2\theta.$$

Reciprocal and Quotient Identities

Definition (Reciprocal Identities)**.**

$$\csc\theta = \frac{1}{\sin\theta}, \qquad \sec\theta = \frac{1}{\cos\theta}, \qquad \cot\theta = \frac{1}{\tan\theta}.$$

Definition (Quotient Identities)**.**

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}.$$

Using Identities in Different Quadrants

Example. Given $\tan\theta = -\frac{3}{4}$ and θ is in quadrant II, find $\sin\theta$ and $\cos\theta$.

Even/Odd and Cofunction Identities

Definition (Even/Odd Identities)**.**

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta \qquad \sec(-\theta) = \sec\theta$$
$$\csc(-\theta) = -\csc\theta \qquad \cot(-\theta) = -\cot\theta$$

Definition (Cofunction Identities)**.**

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$
$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$
$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta \qquad \csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$

Simplifying Expressions

Example. Simplify $\dfrac{\sin\theta}{\tan\theta}$ to a form without quotients.

Verifying Identities

Example. Verify the identity:

$$\tan\theta + \cot\theta = \sec\theta \csc\theta.$$