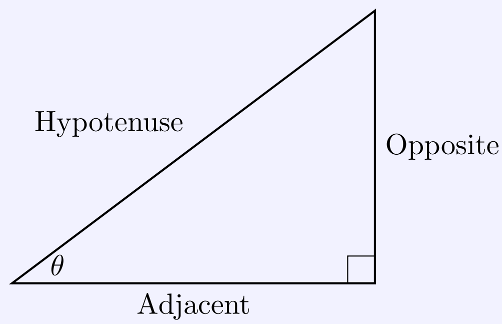
Right Triangle Trigonometry

Theorem. Consider a right triangle with an acute angle θ as shown below. The six trigonometric functions can be defined in terms of the sides of the triangle as follows:



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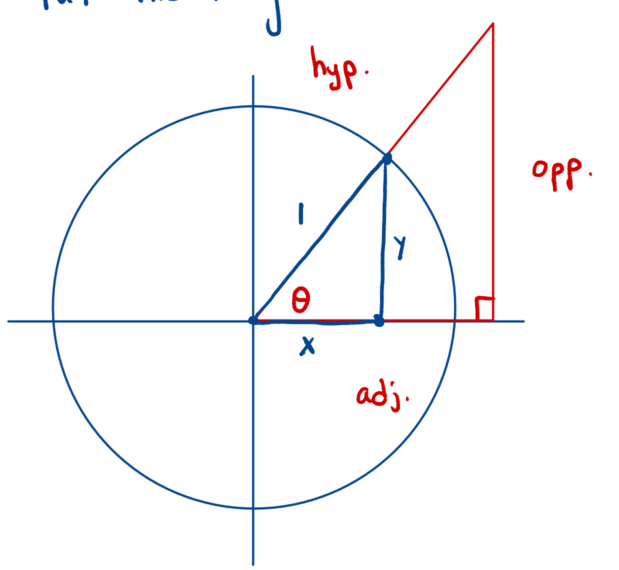
$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \csc(\theta) = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Opposite}}$$

$$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \sec(\theta) = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$$

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}} \qquad \cot(\theta) = \frac{\cos\theta}{\sin\theta} = \frac{\text{Adjacent}}{\text{Opposite}}$$

Put the triangle in the unit circle:



By similar triangles:

if two triangles have the same angles, then the ratios of corresponding sides are the same

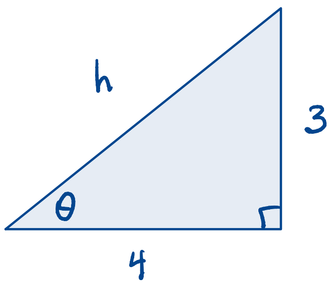
$$\frac{\text{opp}}{\text{hyp}} = \frac{y}{1} = \sin\theta$$

$$\frac{\text{adj}}{\text{hyp}} = \frac{x}{1} = \cos\theta$$

$$\frac{\text{opp}}{\text{adj}} = \frac{y}{x} = \tan\theta$$

Evaluating Trigonometric Functions

Example. Given a right triangle with an angle θ, suppose the side opposite θ has length 3 and the adjacent side has length 4. Evaluate all six trigonometric functions at θ.



$\sin\theta = \frac{o}{h} = \frac{3}{5}$ $\qquad$ $\csc\theta = \frac{5}{3}$

$\cos\theta = \frac{a}{h} = \frac{4}{5}$ $\qquad$ $\sec\theta = \frac{5}{4}$

$h^2 = 3^2 + 4^2$

$h^2 = 9 + 16 = 25$

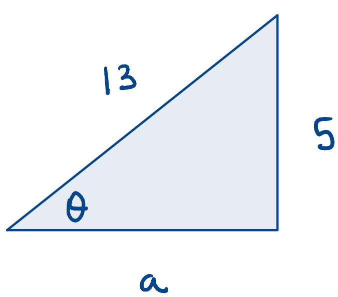
$h = 5$

$\tan\theta = \frac{o}{a} = \frac{3}{4}$ $\qquad$ $\cot\theta = \frac{4}{3}$

Using a Given Trigonometric Value

Example. If $\sin\theta = \frac{5}{13}$ for an acute angle θ in a right triangle, find the remaining five trigonometric functions.

Since $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, so let's draw a triangle with opp $= 5$ and hyp $= 13$.



$\sin\theta = \frac{5}{13}$ (given) $\qquad$ $\csc\theta = \frac{13}{5}$

$\cos\theta = \frac{a}{h} = \frac{12}{13}$ $\qquad$ $\sec\theta = \frac{13}{12}$

$a^2 + 5^2 = 13^2$

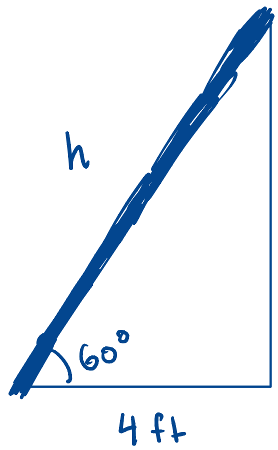
$a^2 = 169 - 25 = 144$

$a = 12$

$\tan\theta = \frac{o}{a} = \frac{5}{12}$ $\qquad$ $\cot\theta = \frac{12}{5}$

Application Example

Example. A ladder leans against a wall, forming a 60° angle with the ground. If the bottom of the ladder is 4 feet from the wall, determine the length of the ladder.



$$\cos 60^\circ = \frac{a}{h} = \frac{4}{h}$$

$$\frac{1}{2} = \frac{4}{h}$$

$$\frac{1}{2}h = 4$$

$$h = 8 \text{ feet}$$