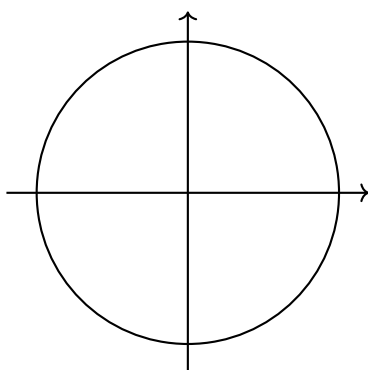


Graphs of Trigonometric Functions

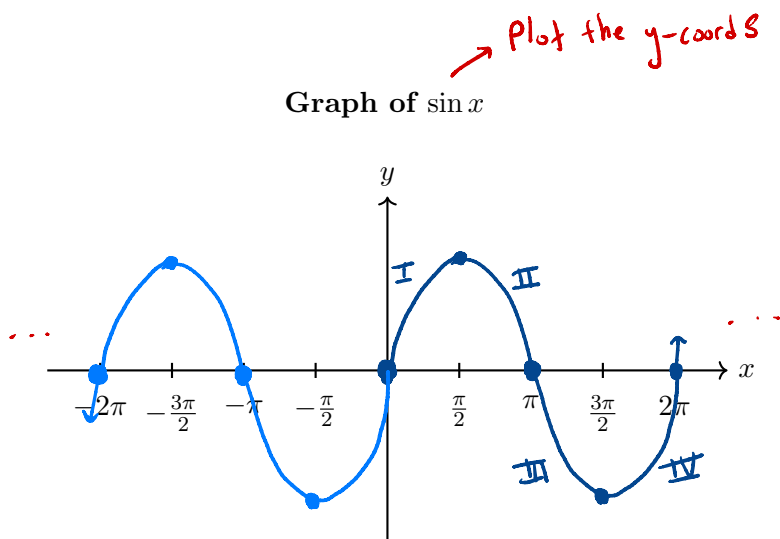
In this section, we will explore the graphs of the six fundamental trigonometric functions: $\sin x$, $\cos x$, $\tan x$, $\sec x$, $\csc x$, and $\cot x$. Traditionally, trigonometric functions are introduced in terms of the angle θ measured in degrees or radians. However, when studying the graphs of these functions, we will use x as the input variable instead of θ . Throughout this section, all angle measurements will be in **radians**, not degrees.

Graph of $\sin x$

Unit Circle



Graph of $\sin x$



Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

Table of Values

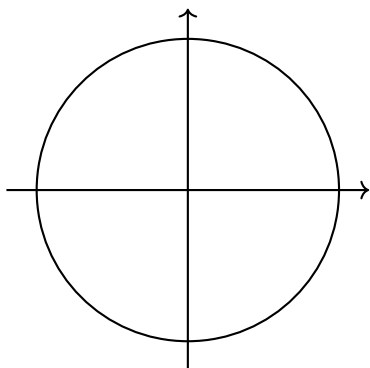
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0

Remark. When we shift or stretch the graphs of trigonometric functions, we can think about these changes in terms of circles that are not centered at the origin or have different sizes.

1. Horizontal and vertical shifts correspond to finding points on a circle that has been shifted sideways or up or down instead of using the usual unit circle centered at the origin.
2. Vertical stretches and compressions are like using a circle with a bigger or smaller radius. Instead of using the unit circle (which has radius 1), we think about points on a circle with radius A , which stretches or shrinks the function's values.
3. Horizontal stretches and compressions affect how quickly the function repeats. This is like changing how fast we move around the circle.

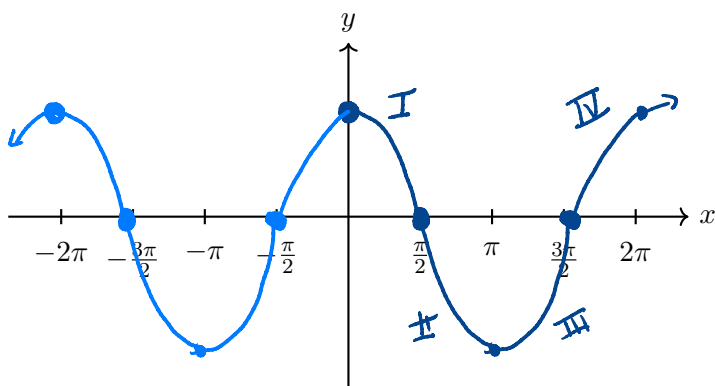
Graph of $\cos x$

Unit Circle



Graph of $\cos x$

Plot the x-coords



Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

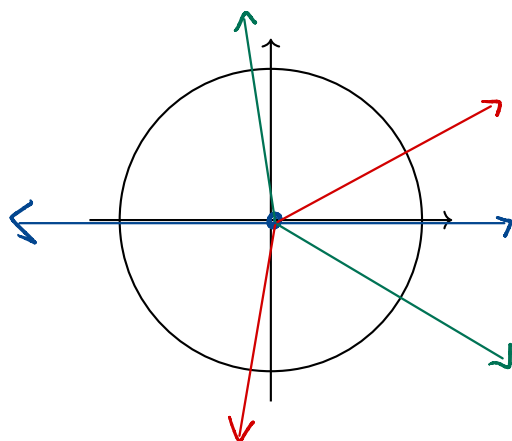
Table of Values

+
I
-
II
-
III
+
IV

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

Graph of $\tan x$

Unit Circle



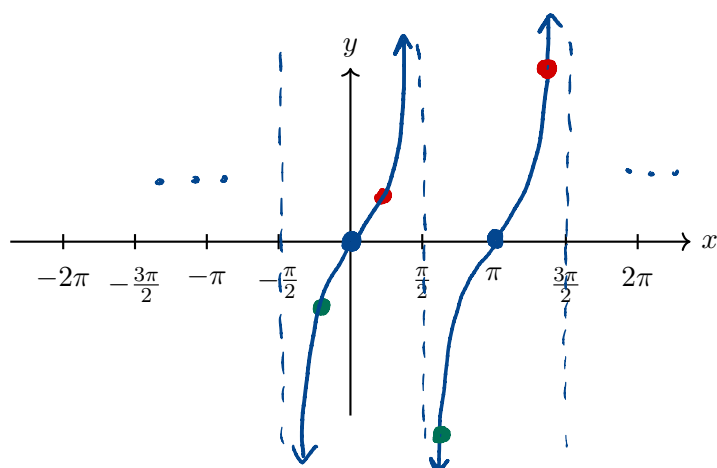
Domain: $x \neq \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}$

Range: $(-\infty, \infty)$

Table of Values

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\tan(x)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	1	undefined	-1	0

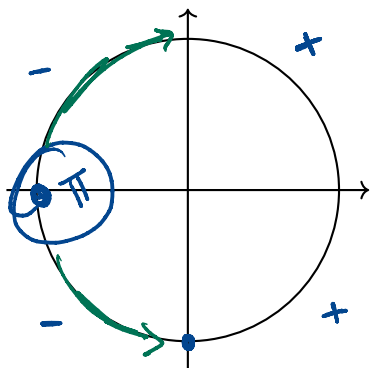
Graph of $\tan x$



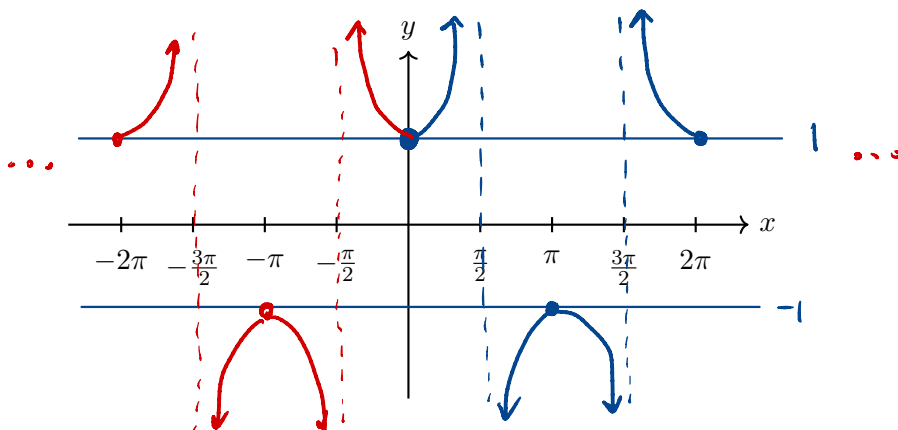
Graph of $\sec x$

$$\sec(x) = \frac{1}{\cos(x)}$$

Unit Circle



Graph of $\sec x$



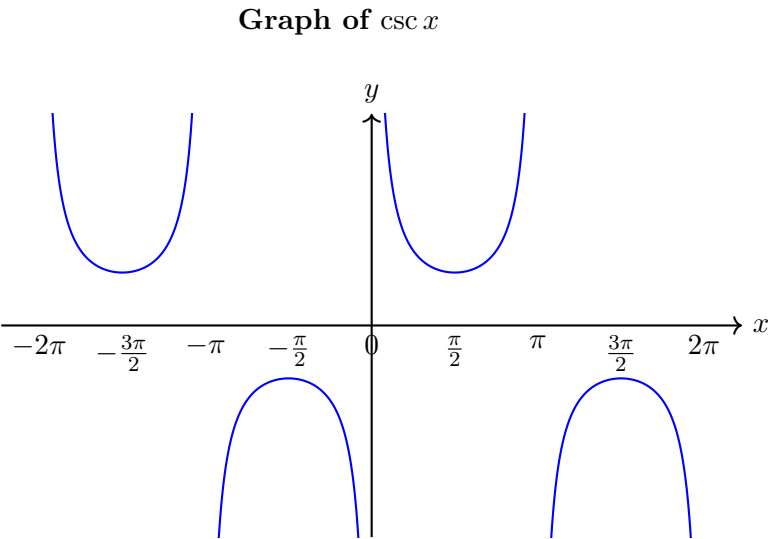
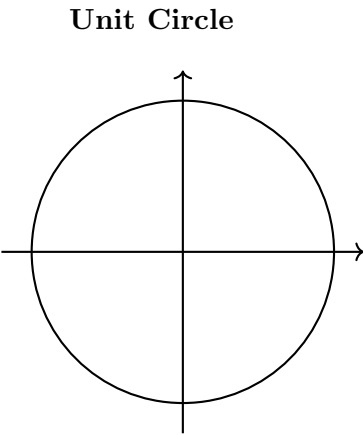
Domain: $x \neq \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}$

Range: $(-\infty, -1] \cup [1, \infty)$

Table of Values

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\sec(x)$	1	$\sqrt{2}$	undefined	$-\sqrt{2}$	-1	$-\sqrt{2}$	undefined	$\sqrt{2}$	1

Graph of $\csc x$



Domain: $x \neq \{k\pi \mid k \in \mathbb{Z}\}$

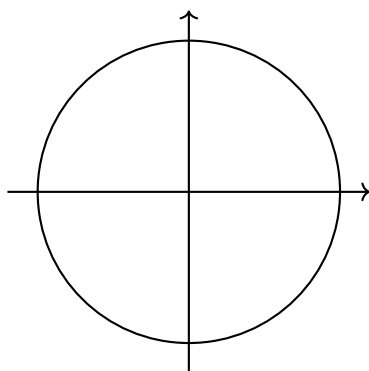
Range: $(-\infty, -1] \cup [1, \infty)$

Table of Values

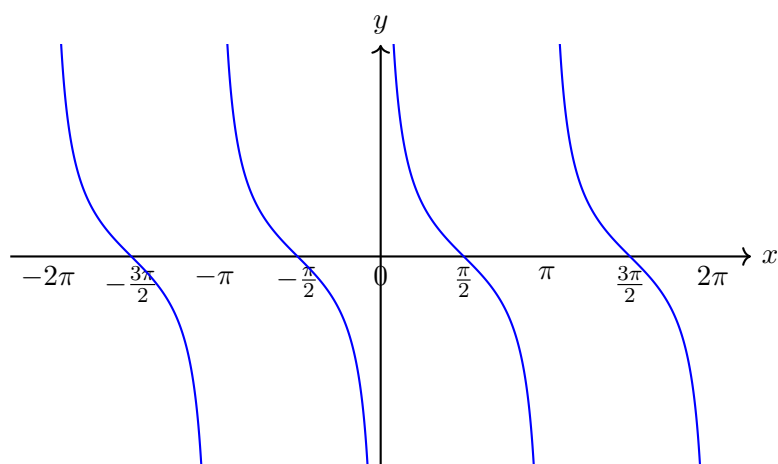
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\csc(x)$	undefined	$\sqrt{2}$	1	$\sqrt{2}$	undefined	$-\sqrt{2}$	-1	$-\sqrt{2}$	undefined

Graph of $\cot x$

Unit Circle



Graph of $\cot x$



Domain: $x \neq \{k\pi \mid k \in \mathbb{Z}\}$

Range: $(-\infty, \infty)$

Table of Values

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\cot(x)$	undefined	1	0	-1	undefined	1	0	-1	undefined