The Other Trigonometric Functions

Previously, we focused on the trigonometric functions:

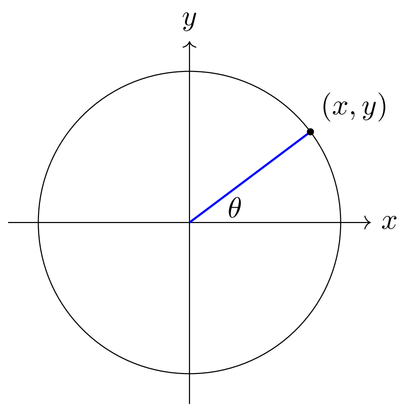
$$\sin\theta \quad \text{and} \quad \cos\theta.$$

Now we expand our study to the other four trigonometric functions:

$$\tan\theta, \quad \sec\theta, \quad \csc\theta, \quad \cot\theta.$$

Finding $\tan\theta$ from the Unit Circle

Recall that when an angle θ is drawn in standard position, the point where its terminal side intersects the unit circle has coordinates $(x, y) = (\cos\theta, \sin\theta)$.



The tangent of an angle θ is defined as

provided $x \neq 0$

Question. Explain why $\tan\theta$ can be interpreted as the slope of the terminal side of the angle θ.

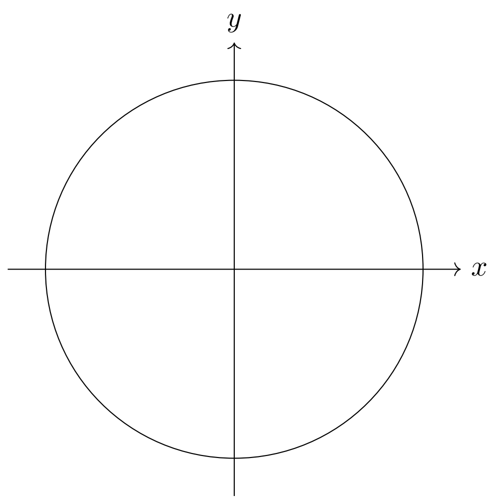
Question. What are the domain and range of $\tan\theta$?

Finding $\sec\theta, \csc\theta$, and $\cot\theta$ from the Unit Circle

Definition. Let θ be an angle in standard position whose terminal side passes through the point (x, y) in the plane, and let $r = \sqrt{x^2 + y^2}$ be the distance from the point to the origin. Then the three additional trigonometric functions are defined by

$$\sec(\theta) = \frac{r}{x}, \quad \csc(\theta) = \frac{r}{y}, \quad \cot(\theta) = \frac{x}{y}.$$

When $r = 1$ (i.e. on the unit circle), these definitions reduce to



Function	Domain	Range
$\sec(\theta)$		$(-\infty, -1] \cup [1, \infty)$
$\csc(\theta)$		$(-\infty, -1] \cup [1, \infty)$
$\cot(\theta)$		$(-\infty, \infty)$

Function	I	II	III	IV
$\sin\theta$				
$\cos\theta$				
$\tan\theta$				
$\cot\theta$				
$\sec\theta$				
$\csc\theta$				

Example. Compute $\tan(225°)$.

Example. Compute $\sec\left(\frac{\pi}{3}\right)$.

Example. Compute $\cot\left(\frac{2\pi}{3}\right)$.