

Sine and Cosine

Definition. For a circle of radius r , the general definitions are

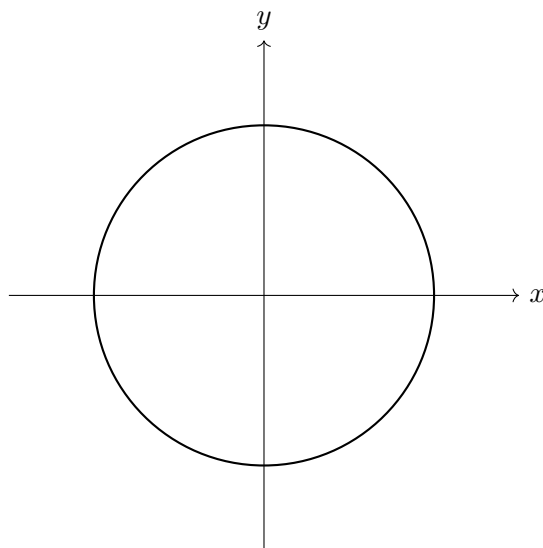
$$\sin(\theta) = \frac{y}{r} \quad \text{and} \quad \cos(\theta) = \frac{x}{r},$$

where (x, y) is a point on the terminal side of the angle θ . When we use the *unit circle*, these definitions simplify to:

$$\sin(\theta) = y \quad \text{and} \quad \cos(\theta) = x.$$

Remark. Changing the circle's radius does not affect the sine and cosine values since the formulas are defined as ratios that cancel out the effect of the radius.

Question. Draw a picture explaining the definitions of sine and cosine using the circle below:



Domain and Range

Question. What are the domains of $\cos(\theta)$ and $\sin(\theta)$?

Question. What are the ranges of $\cos(\theta)$ and $\sin(\theta)$?

Evaluating Sine and Cosine using Reference Angles

A *reference angle* is the acute angle between the terminal side of θ and the horizontal axis. The following relationship holds for a trigonometric function T (where T can be sine or cosine):

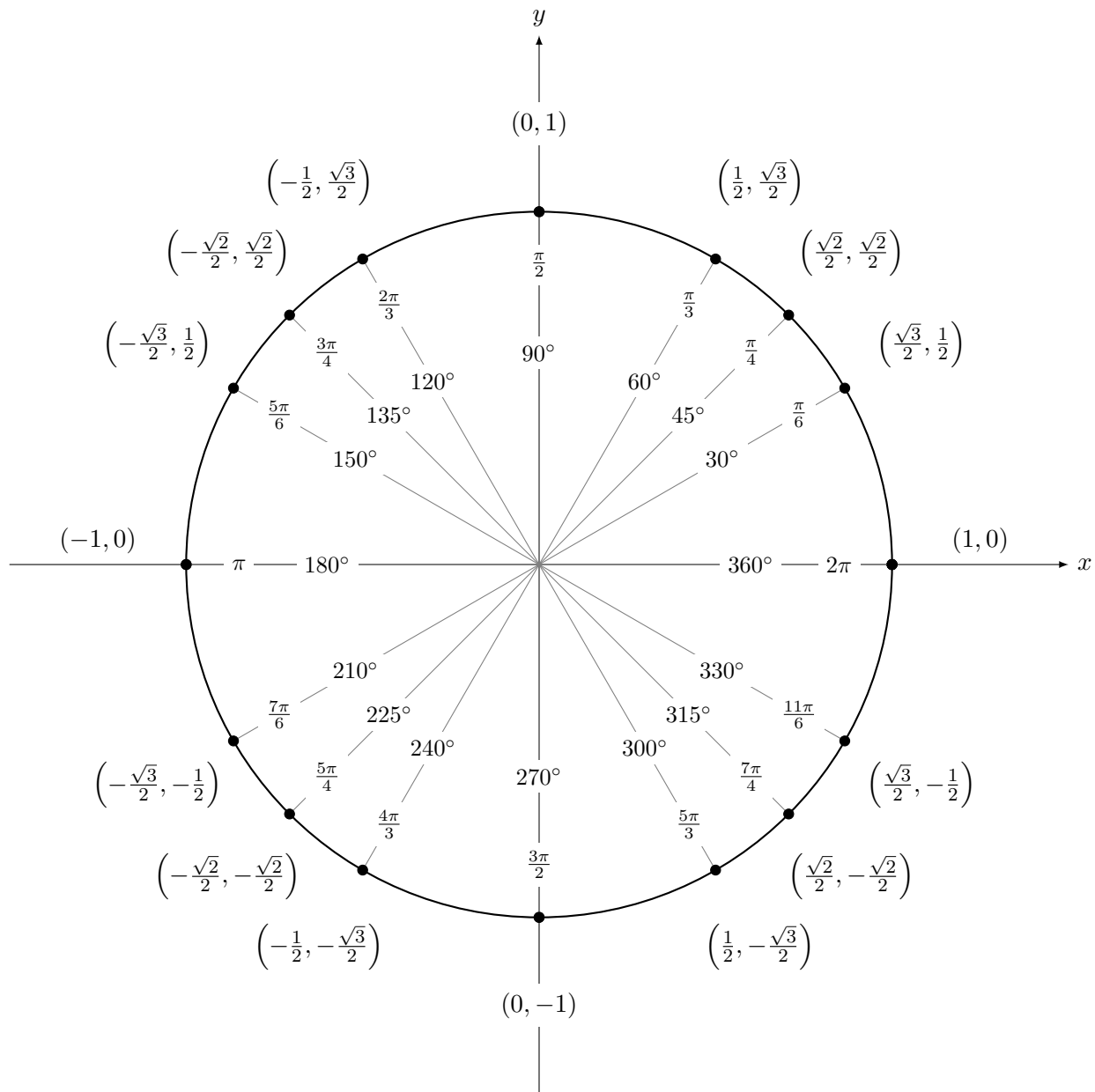
$$T(\theta) = \pm T(\theta_{\text{ref}}),$$

with the sign determined by the quadrant in which θ lies.

Example. Evaluate $\sin(150^\circ)$ using reference angles.

Example. Evaluate $\cos(225^\circ)$ using reference angles.

Example. Evaluate $\sin\left(\frac{11\pi}{6}\right)$ using reference angles.



Angle (deg)	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
Angle (rad)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0