Sine and Cosine

Definition. For a circle of radius r, the general definitions are

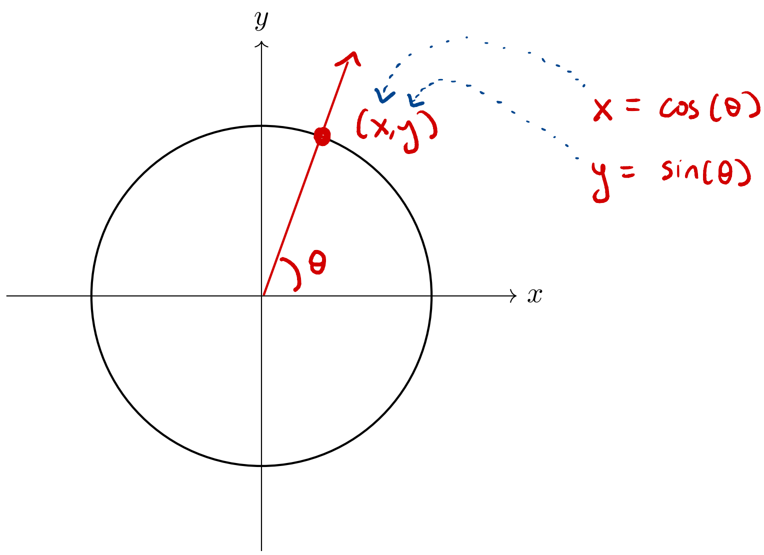
$$\sin(\theta) = \frac{y}{r} \quad \text{and} \quad \cos(\theta) = \frac{x}{r},$$

where (x, y) is a point on the terminal side of the angle θ. When we use the *unit circle*, these definitions simplify to:

$$\sin(\theta) = y \quad \text{and} \quad \cos(\theta) = x.$$

Remark. Changing the circle's radius does not affect the sine and cosine values since the formulas are defined as ratios that cancel out the effect of the radius.

Question. Draw a picture explaining the definitions of sine and cosine using the circle below:



Domain and Range

Question. What are the domains of $\cos(\theta)$ and $\sin(\theta)$?

Every angle θ corresponds to a point on the unit circle. Domain $(-\infty, \infty)$

Question. What are the ranges of $\cos(\theta)$ and $\sin(\theta)$?

The x and y coordinates on the unit circle are bounded between −1 and 1. Range : $[-1, 1]$

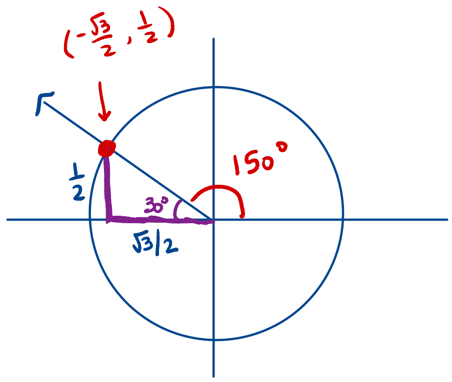
Evaluating Sine and Cosine using Reference Angles

A *reference angle* is the acute angle between the terminal side of θ and the horizontal axis. The following relationship holds for a trigonometric function T (where T can be sine or cosine):

$$T(\theta) = \pm T(\theta_{\text{ref}}),$$

with the sign determined by the quadrant in which θ lies.

Example. Evaluate $\sin(150°)$ using reference angles.

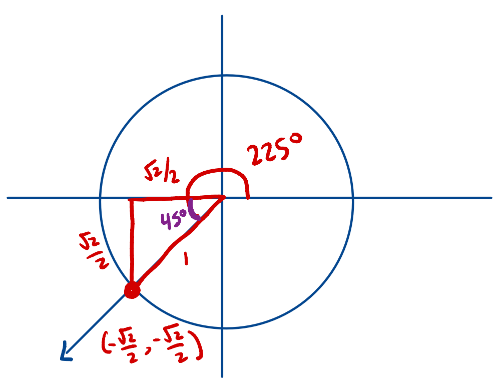


$\theta_{ref} = 180° - 150° = 30°$

$\sin(150°) = \frac{1}{2}$

$\cos(150°) = -\frac{\sqrt{3}}{2}$

Example. Evaluate $\cos(225°)$ using reference angles.

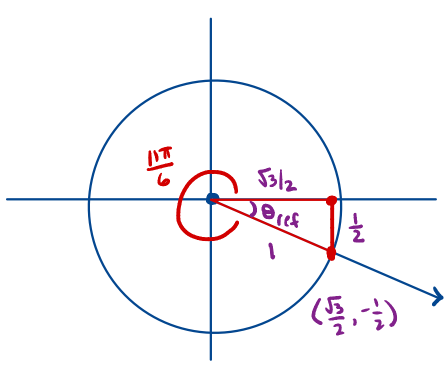


$\theta_{ref} = 225° - 180° = 45°$

$\cos(225°) = -\frac{\sqrt{2}}{2}$

$\sin(225°) = -\frac{\sqrt{2}}{2}$

Example. Evaluate $\sin\left(\frac{11\pi}{6}\right)$ using reference angles.

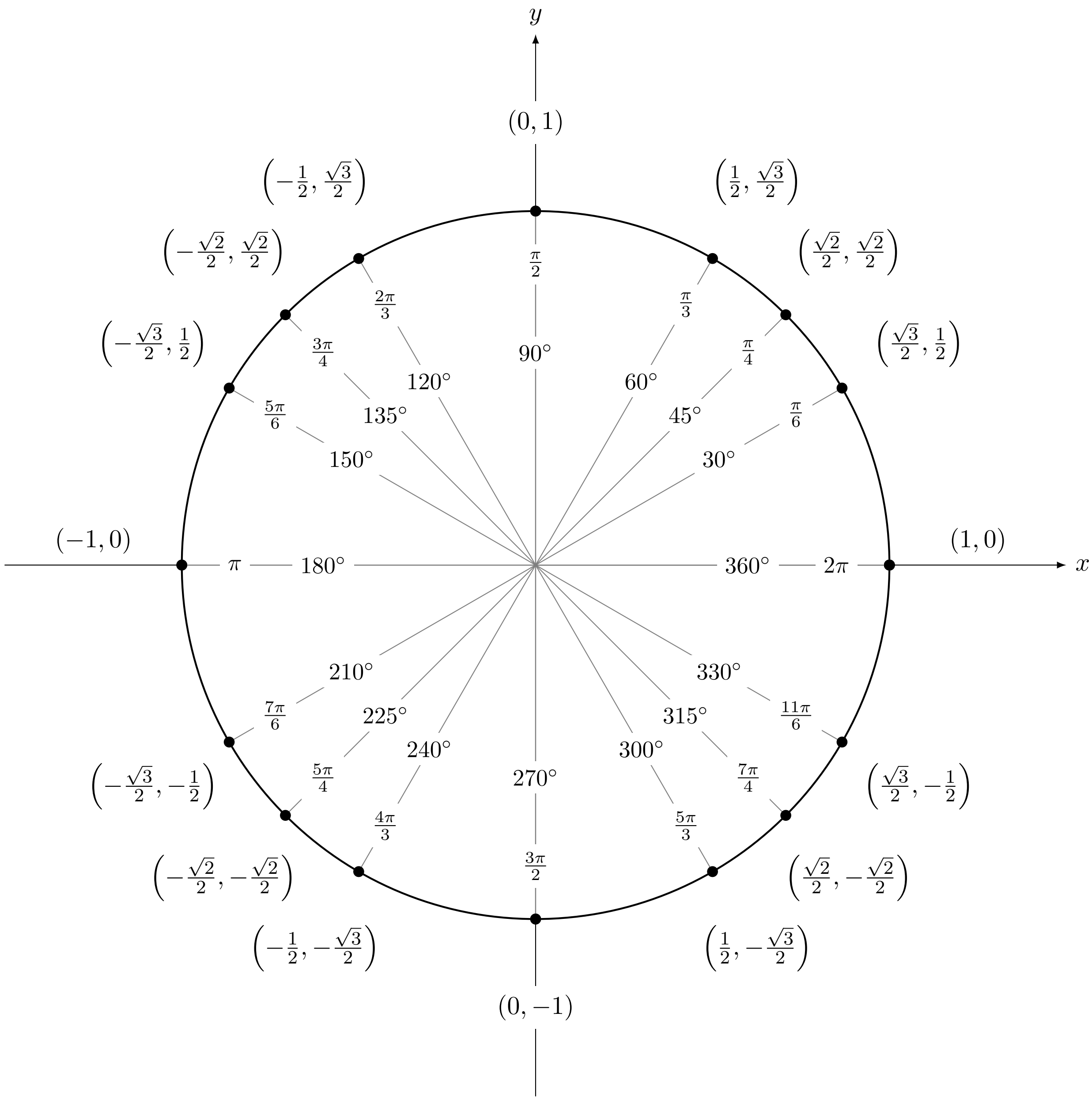


$30°$

$\theta_{ref} = 2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$

$\cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}$



Angle (deg)	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
Angle (rad)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0