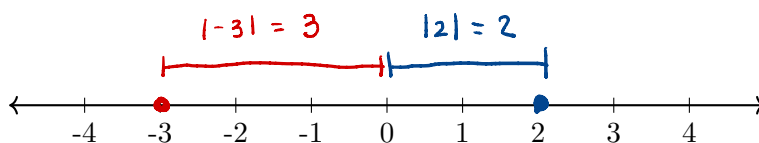


Absolute Values

Geometric Definition of Absolute Value

Definition. What is the absolute value of a number x ?



The absolute value of a number x , written $|x|$, is the distance from x to zero on the number line.

* Absolute values are always non-negative (distances cannot be negative).

Piecewise Definition of Absolute Value

Although not critical now, the piecewise definition introduces conditional formulas and previews piecewise functions, which we will see in Week 2.

Definition.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Example. Evaluate $|x|$ using the piecewise definition for $x = 5$ and $x = -7$.

Because $5 \geq 0$, we use the first equation $|x| = x$. So $|5| = 5$

Because $-7 < 0$, we use the second equation $|x| = -x$. So $|-7| = -(-7) = 7$

Solving Absolute Value Equations

e.g. Solve $|x - 4| = -5$

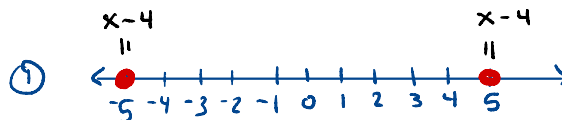
General Method: To solve $|A| = B$

Case 1: If $B < 0$, there is **no solution**. *← Because absolute values cannot be negative*

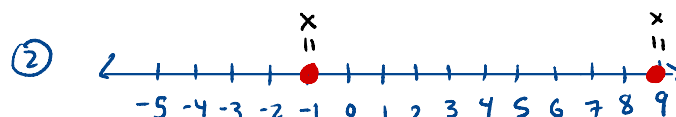
Case 2: If $B \geq 0$, split into two cases: $A = B$ or $A = -B$.

Example. Solve $|x - 4| = 5$:

① $x - 4 = 5$ OR $x - 4 = -5$



② $x = 9$ OR $x = -1$



Solving Absolute Value Inequalities

General Method: Absolute Value Inequalities

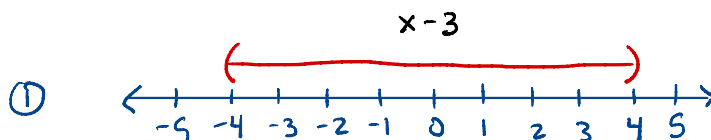
Case 1: $|A| < B$: Rewrite as $-B < A < B$.

Case 2: $|A| > B$: Rewrite as $A > B$ or $A < -B$.

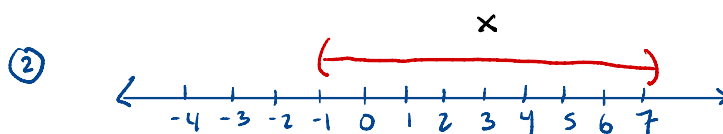
(if B is positive these are the only cases)

(Case 1) **Example.** Solve $|x - 3| < 4$.

① $-4 < x - 3 < 4$



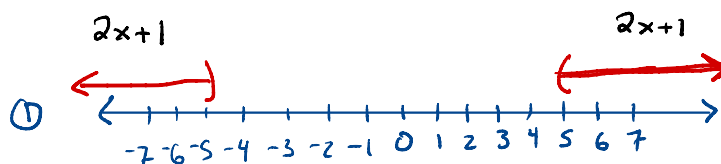
② $-1 < x < 7$



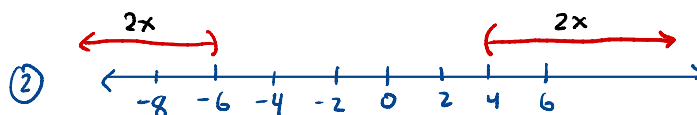
$(-1, 7)$

(Case 2) **Example.** Solve $|2x + 1| > 5$.

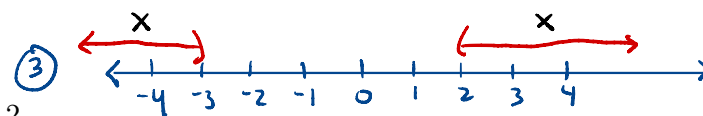
① $2x + 1 > 5$ OR $2x + 1 < -5$



② $2x > 4$ OR $2x < -6$



③ $x > 2$ OR $x < -3$



$(-\infty, -3) \cup (2, \infty)$

Application: Manufacturing

Absolute values are used to define acceptable ranges for product dimensions.

Example. Suppose part needs to be 5 cm long, with a tolerance of ± 0.1 cm. The acceptable range for the length x of the part is:

$$|x - 5| \leq 0.1$$

Inequalities like this ensure consistency in manufacturing industries like automotive, aerospace, and electronics.

