

Angles, Radians, Area, and Arc Length

Angles as Measures of Rotation

Definition. An **angle** is defined as the measure of rotation between two rays (sides) sharing a common endpoint (vertex).

- **Initial Side:** the fixed, starting side.
- **Terminal Side:** the side that rotates about the vertex.

Definition. Angle measures can be positive or negative depending on the direction of rotation:

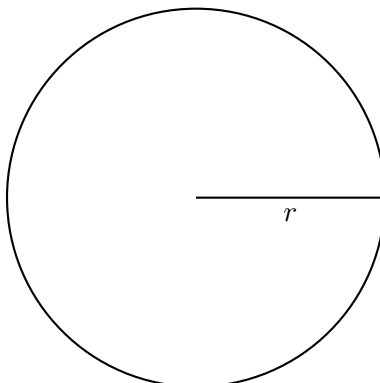
- Positive angles are defined by rotations in the _____ direction.
- Negative angles are defined by rotations in the _____ direction.

Units for Measuring Angles

Definition. Angles are measured in two primary units:

- Degrees: One complete rotation is _____ .
- Radians: A radian is defined as _____ at the center of a circle by an arc whose length is equal to the radius of the circle. One full rotation equals _____ .

Example. Draw rotations measuring 1, 2, and 3 radians.



Since a full circle is 360° and also 2π radians, we derive the conversion factors:

$$1 \text{ radian} = \frac{180^\circ}{\pi}, \quad 1^\circ = \frac{\pi}{180} \text{ radians.}$$

Example. Convert 45° to radians.

Example. Convert $\frac{5\pi}{6}$ radians to degrees.

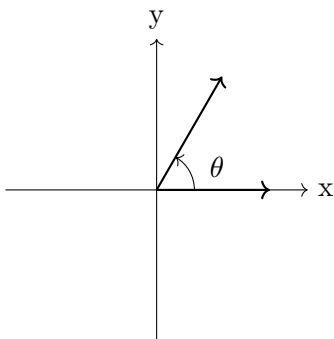
Example. Convert 120° to radians.

Example. Convert $\frac{7\pi}{4}$ radians to degrees.

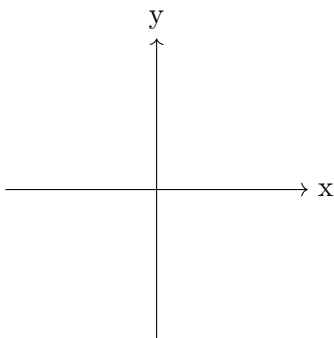
The Coordinate Plane

Definition. An angle is in **standard position** if its vertex is _____ and

its initial side lies along the _____ .



Definition. The Cartesian plane is divided into four quadrants:



Coterminal Angles

Definition. Coterminal angles _____ but

_____ .

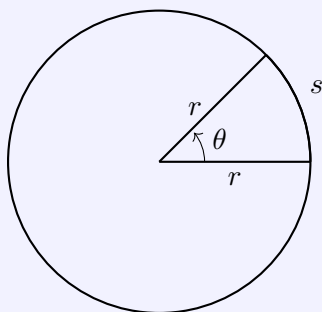
Example. Find an angle coterminal with 30° .

Arc Length

Definition. The **arc length** of a circle is the distance along the curved line forming the arc. The arc length s can be computed using:

- **Radians:** $s = r\theta$
- **Degrees:** $s = \frac{\pi r\theta}{180}$

where θ is the central angle in either radians or degrees, respectively.



Example. Find the length of an arc in a circle with radius 5 cm and central angle 60° .

Example. Find the length of an arc in a circle with radius 10 m and central angle $\frac{\pi}{4}$ radians.

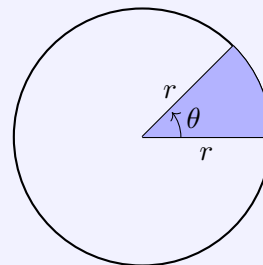
Example. A circle has an arc length of 8π cm and a radius of 4 cm. Find the central angle in radians.

Example. A circle has an arc length of 20 cm and a central angle of 90° . Find the radius.

Area of a Sector

Definition. The **sector area** A subtended by a central angle θ in a circle of radius r is given by:

$$A = \begin{cases} \frac{1}{2}r^2\theta & \text{(radians)} \\ \frac{\pi r^2\theta}{360} & \text{(degrees)} \end{cases}$$



□

Example. Find the area of a sector in a circle with radius 6 cm and a central angle of 120° .

Example. Find the area of a sector in a circle with radius 10 m and a central angle of $\frac{\pi}{3}$ radians.

Example. A sector in a circle with radius 8 cm has an area of $32\pi \text{ cm}^2$. Find the central angle in radians.

Example. A sector has an area of $25\pi \text{ cm}^2$ and a central angle of 90° . Find the radius of the circle.