

Properties of Logarithms

Product Rule:

$$\log_b(A \cdot B) = \log_b(A) + \log_b(B)$$

Quotient Rule:

$$\log_b\left(\frac{A}{B}\right) = \log_b(A) - \log_b(B)$$

Power Rule:

$$\log_b(A^c) = c \log_b(A)$$

Example. Simplify $\log(5^3)$.

Example. Evaluate $\log_2(8^3)$.

Example. Evaluate $\log_3\left(\frac{81}{9}\right)$.

Example. Expand $\log\left(\frac{x^2}{y^3}\right)$.

Example. Combine $\log_3(15) - \log_3(40) + \log_3(72)$ into a single logarithm and simplify.

Change of Base Formula: For any positive A and bases $b, c > 0$ (with $b \neq 1$ and $c \neq 1$),

$$\log_b(A) = \frac{\log_c(A)}{\log_c(b)}$$

Example. Simplify $\log_5(14)$ using base 10 and base e .

Example. Evaluate $\log_2(64^3)$ without directly computing 64^3 .

Example. Expand $\log(\sqrt{x^2 + 2})$.

Example. Expand the logarithm:

$$\log\left(\frac{x^3\sqrt{y}}{(z^2)^3}\right).$$

Example. Combine and simplify the following expression into a single logarithm:

$$2 \log(x) - \frac{1}{2} \log(y) + \log(5).$$

Example. Evaluate the expression:

$$\log_2\left(\frac{16^3}{8^2}\right).$$