Logarithmic Functions

Definition. Let a be a positive real number such that $a \neq 1$. The **logarithmic function** with base a is defined by

$$f(x) = \log_a(x),$$

for all $x > 0$. This function is the inverse of the exponential function a^x. In other words, for any $x > 0$ and real number y, we have

$$y = \log_a(x) \quad \Longleftrightarrow \quad a^y = x.$$

Evaluating Logarithmic Expressions

Example. Evaluate the expression $\log_2(8)$.

Example. Evaluate the following expressions.

Problem	**Answer**
$\log_3(81)$	
$\log_{10}(1000)$	
$\log_5(\frac{1}{5})$	
$\log_4(2)$	

Graphing Logarithmic Functions

The graph of the logarithmic function $f(x) = \log_a(x)$ (with $a > 1$) has several key features that help us understand its behavior:

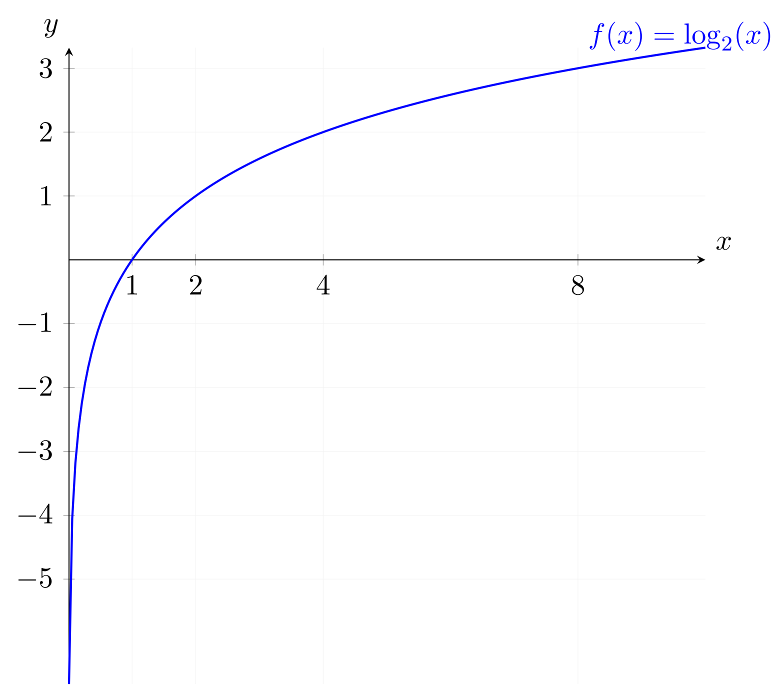
- **Domain:**

- **Range:**

- **Vertical Asymptote:**

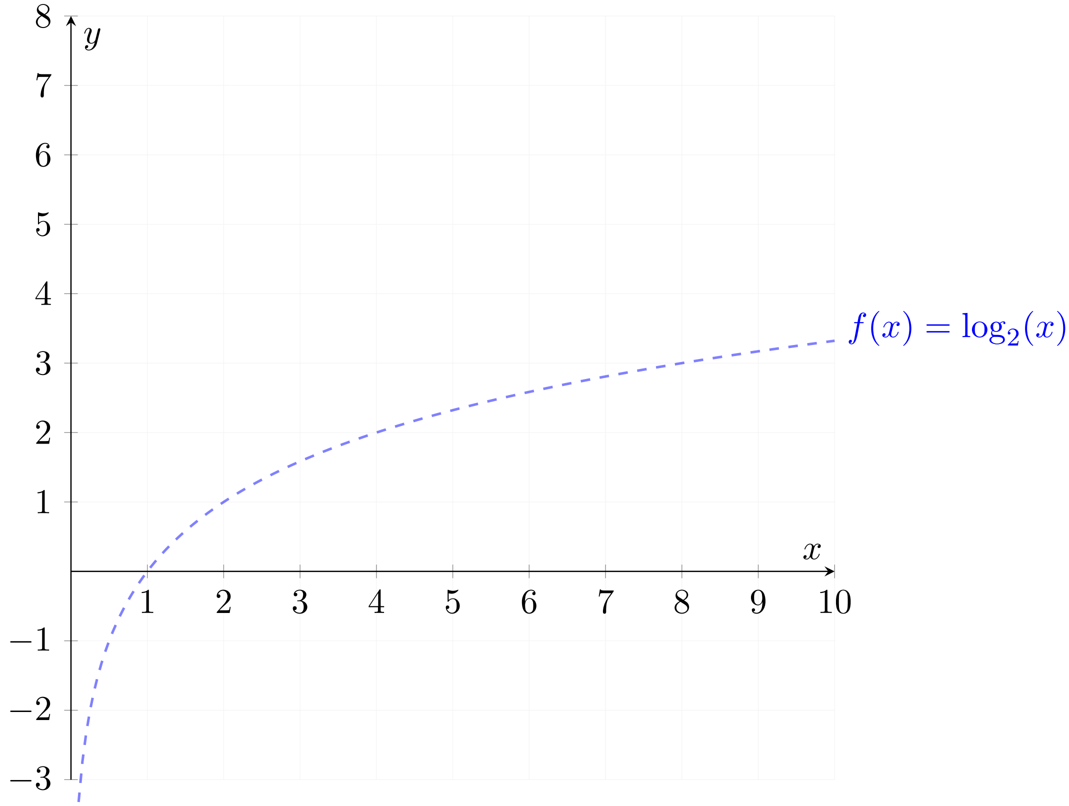
- **End Behavior:**

Below is an example graph of the function $f(x) = \log_2(x)$ to illustrate these features:



Graph Transformations

Example. Graph the function $g(x) = \log_2(x - 1) + 2$.



Common and Natural Logarithms

Definition.

- The **common logarithm** is the logarithm with base 10:

$$\log(x) = \log_{10}(x).$$

- The **natural logarithm** is the logarithm with base e (where $e \approx 2.71828$):

$$\ln(x) = \log_e(x).$$

Finding Domains of Logarithmic Functions

Example. Determine the domain of the function $f(x) = \ln(4 - x^2)$, express your answer in interval notation.

Example. Determine the domain of the function $g(x) = \log_3(2x + 1)$, and express your answer in interval notation.

Inverses of Exponential and Logarithmic Functions

Exponential and logarithmic functions are inverses of each other. In general,

$$f(x) = a^x \quad \Longleftrightarrow \quad f^{-1}(x) = \log_a(x).$$

Example. Find the inverse of $f(x) = 3^x$.

Example. Find the inverse of $g(x) = \log_5(x)$.