# Graphing Polynomials and the Fundamental Theorem of Algebra

## Introduction

Polynomials are one of the most fundamental objects in mathematics, appearing in algebra, calculus, and applied sciences. In this lecture, we will explore the properties and behaviors of polynomial functions, focusing on their definitions, roots, and graphical representations.

## Defining Polynomials

A polynomial function is defined as a function of the form:

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where:

- $a_n, a_{n-1}, \ldots, a_0$ are constants (coefficients),
- $n$ is a non-negative integer (the degree of the polynomial),
- $a_n \neq 0$ (the leading coefficient).

**Example.** Describe the polynomial $P(x) = 2x^3 - 5x^2 + x - 7$.

## The Fundamental Theorem of Algebra

**Theorem** (The Fundamental Theorem of Algebra)**.** Every non-zero polynomial of degree $n$ with complex coefficients has exactly $n$ roots in the complex number system (counting multiplicities).

**Question.** What does the Fundamental Theorem of Algebra imply?

**Example.** Find the roots of $P(x) = x^3 - 4x$.

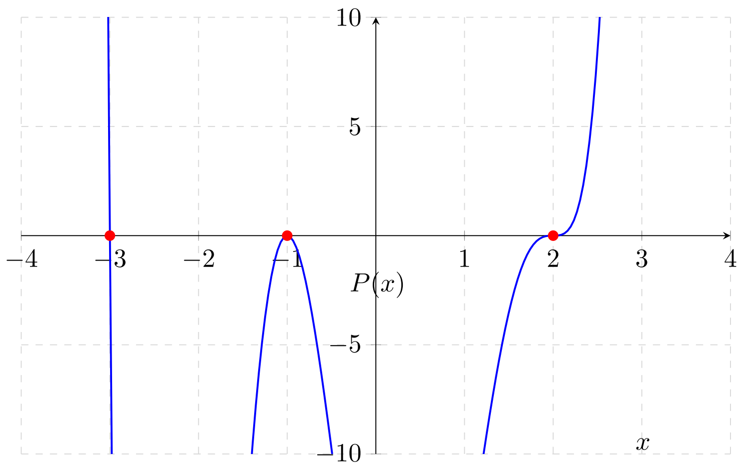
## Behavior of Polynomials at Zeros and Multiplicities

The behavior of a polynomial near its zeros is determined by the multiplicity of each zero. Here is how to compute and interpret multiplicity.

**Definition.** What is the multiplicity of a zero $x = c$?

**Question.** How to interpret multiplicity graphically?

**Example.** Analyze the polynomial $P(x) = (x+1)^2(x-2)^3(x+3)$.



## End Behavior of Polynomials

The end behavior of a polynomial is determined by its leading term (the term with the highest power of $x$).

**Example.** Determine the end behavior of $P(x) = 3x^4 - 5x^3 + x - 7$ using the formal method.

**Example.** Determine the end behavior of $P(x) = -2x^3 + 5x^2 - x + 1$ using the leading term.

## Graphing Factored Polynomials with a Sign Chart

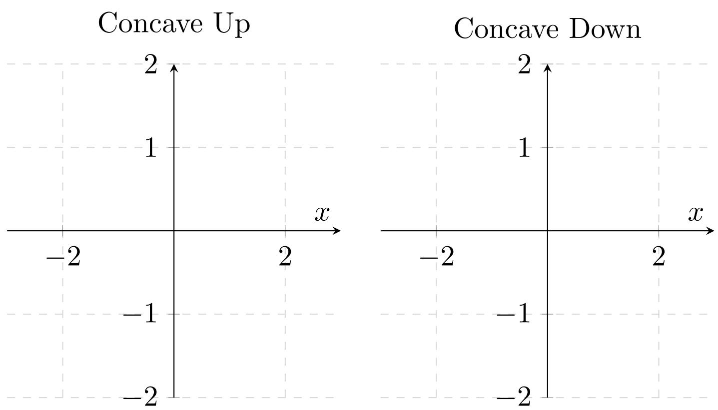
A number line sign chart helps determine the sign of $P(x)$ between its zeros.

**Example.** Graph $P(x) = (x-1)(x+2)^2$ using a sign chart.

## Understanding Concavity in Polynomials

Concavity describes how a curve bends:

- **Concave up:** The graph looks like a cup holding water.
- **Concave down:** The graph looks like a cup spilling water.




**Example.** Analyze the concavity of $P(x) = x^3 - 3x^2$.