

Factoring Polynomials

Introduction

Factoring polynomials is an essential skill in algebra that simplifies expressions and solves equations. In this lecture, we will review methods of factoring, including factoring out the greatest common factor and factoring differences of squares.

Factoring Out the Greatest Common Factor (GCF)

The greatest common factor (GCF) of a polynomial is the largest factor shared by all terms in the expression. Factoring out the GCF simplifies the polynomial for further operations.

Example. Factor $6x^3 + 9x^2 - 12x$.

Find the GCF of all terms: $3x$

Factor out $3x$: $3x(2x^2 + 3x - 4)$

Example. Factor $15x^5y^3 - 10x^3y^2 + 5x^2y$.

Find the GCF of all terms: $5x^2y$

Factor out $5x^2y$: $5x^2y(3x^3y^2 - 2xy + 1)$

Factoring Differences of Squares

The difference of squares is a special factoring pattern:

$$a^2 - b^2 = (a - b)(a + b).$$

This applies whenever two terms are perfect squares and are subtracted.

Example. Factor $x^2 - 16$.

This is a difference of squares

$$x^2 - 16 = (x - 4)(x + 4)$$

Example. Factor $4x^2 - 25y^2$.

$$4x^2 - 25y^2 = (2x - 5y)(2x + 5y)$$

Example. Factor $9x^4 - 1$.

$$9x^4 - 1 = (3x^2 - 1)(3x^2 + 1)$$

The Rational Root Theorem

The Rational Root Theorem provides a method to identify potential rational roots of a polynomial equation. While not a primary focus, it can be useful for solving higher-degree polynomials.

Example. Find the possible rational roots of $2x^3 - 3x^2 + x - 6 = 0$.

Rational Root Thm: potential roots are of the form $\pm \frac{p}{q}$
where p divides the constant term and q divides the leading term.

Constant term: 6 Factors: 1, 2, 3, 6

Leading coefficient: 2 Factors: 1, 2

Possible Rational Roots: $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm 6, \pm \frac{3}{2}$

Applications of Factoring Polynomials

Factoring polynomials is a foundational skill with applications across various fields, including:

- **Solving Equations:** Factoring simplifies polynomial equations, making it easier to find roots or solutions.
- **Physics:** Models motion, forces, and energy, where factoring helps find critical points such as maximum height or equilibrium.
- **Economics:** Analyzes profit, cost, and revenue functions to identify breakeven points.
- **Optimization:** In calculus, factoring aids in solving problems involving maxima, minima, and inflection points.