Algebra Review

Simplifying Fractional Expressions with Exponents

Rule Name	Formula
Product Rule	$a^m \cdot a^n = a^{m+n}$
Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power	$(a^m)^n = a^{m \cdot n}$
Negative Exponent	$a^{-n} = \frac{1}{a^n}, \; a \neq 0$
Fractional Exponent	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Example. Simplify $\dfrac{x^5}{x^2}$.

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

Example. Simplify $\dfrac{3x^{-2}}{9x^{-4}}$.

$$\frac{3x^{-2}}{9x^{-4}} = \frac{3}{9} \cdot \frac{x^{-2}}{x^{-4}} = \frac{1}{3} \cdot x^{-2-(-4)} = \frac{1}{3} x^2$$

Example. Simplify the expression $\dfrac{a^{-3}b^5}{a^2b^{-4}}$, eliminating negative exponents.

$$\frac{a^{-3}b^5}{a^2 b^{-4}} = a^{-3-2} \cdot b^{5-(-4)} = a^{-5} \cdot b^9 = \frac{b^9}{a^5}$$

Solving Non-Linear Inequalities

Steps to Solve

1. Write the expression in factored form (if not already factored).
2. Find the critical points by setting each factor equal to zero.
3. Test the intervals between critical points.

Example. Solve $x^2 - x - 2 > 0$.

1 Factor :

$$x^2 - x - 2 = (x-2)(x+1) > 0$$

2 Find the critical points (where the expression is 0)

$$x - 2 = 0 \Rightarrow x = 2$$

$$x + 1 = 0 \Rightarrow x = -1$$

3

Interval	$x-2$	$x+1$	$(x-2)(x+1)$
$(-\infty, -1)$	$-$	$-$	$+$
$(-1, 2)$	$-$	$+$	$-$
$(2, \infty)$	$+$	$+$	$+$

$$\boxed{(-\infty, -1) \cup (2, \infty)}$$

Example. Solve $\dfrac{x-6}{x+1} \geq 0$.

① Both numerator and denominator are factored ✓

② Find the critical points by setting the numerator and denominator equal to 0:

$x - 6 = 0 \Rightarrow x = 6$ (fraction is 0 here)

$x + 1 = 0 \Rightarrow x = -1$ (undefined here)

③

Interval	$x-6$	$x+1$	$(x-6)/(x+1)$
$(-\infty, -1)$	$-$	$-$	$+$
$(-1, 6)$	$-$	$+$	$-$
$(6, \infty)$	$+$	$+$	$+$

$$\boxed{(-\infty, -1) \cup [6, \infty)}$$

Simplifying Radicals and Fractional Exponents

Example. Simplify $\sqrt[5]{32x^{10}y^{15}}$.

$$\sqrt[5]{32\,x^{10}\,y^{15}} = 32^{1/5} \cdot (x^{10})^{1/5} \cdot (y^{15})^{1/5}$$

$$= 2 \cdot x^2 \cdot y^3$$

Example. Simplify $\sqrt[4]{a^3b} \cdot \sqrt[4]{16a^5b^3}$.

Remember: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$

We get

$$\sqrt[4]{(a^3b) \cdot (16a^5b^3)}$$

$$= \sqrt[4]{16\,a^{3+5}\,b^{1+3}}$$

$$= \sqrt[4]{16\,a^8\,b^4}$$

$$= \sqrt[4]{16} \cdot \sqrt[4]{a^8} \cdot \sqrt[4]{b^4}$$

$$= 2 \cdot a^2 \cdot b$$