Midterm 2

Differential Geometry of Curves and Surfaces
MATH 4230/5230
Fall 2026

Friday, October 30, 2026

PRACTICE EXAM

NAME: _____

1 problem; 100 points total.

Problem 1: 100 points.

Problem:	1	Total
Possible:	100	100
Earned:		

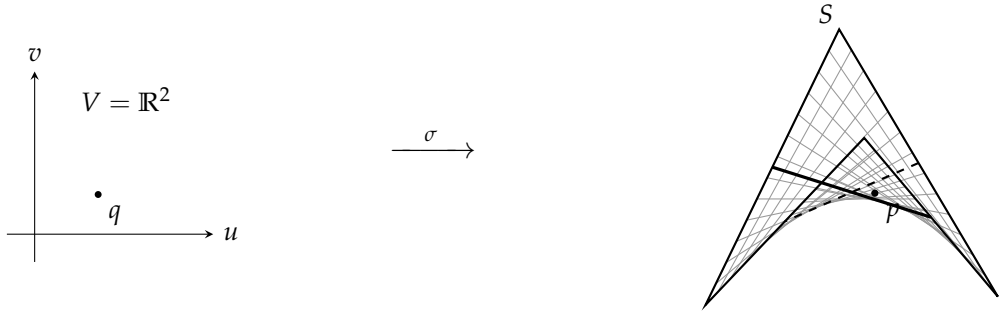
- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete the exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam as a single **.pdf** to **Canvas**, with this **cover sheet** and the questions in the correct order.
- You have 45 minutes to complete the exam.

Problem 1 **100 points**

Let $S \subseteq \mathbb{R}^3$ be a regular oriented surface with coordinate patch

$$\sigma(u, v) = (u, v, uv), \qquad (u, v) \in V = \mathbb{R}^2.$$

We will use the orientation of S determined by σ. Let $q = (u, v)$ and $p = \sigma(q)$.



$$q = (u, v) \qquad \longmapsto \qquad p = \sigma(q)$$

You may use without proof that the Jacobian matrix is given by

$$J(\sigma) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ v & u \end{bmatrix}.$$

Problem 1, continued.

1(a) **(15 points)** *Use the cross product to derive the following formula for the Gauss map*

$$N : S \longrightarrow S^2 \subseteq \mathbb{R}^3$$

in the given local coordinates:

$$N(\sigma(u,v)) = \frac{(-v,-u,1)}{\sqrt{1+u^2+v^2}}.$$

Recall that $\sigma(u,v) = (u,v,uv)$, and we are using the orientation given by σ.

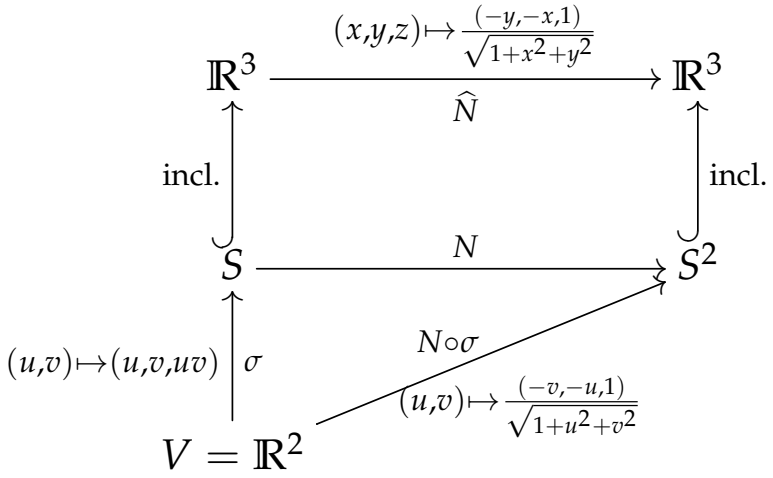
Page subtotal: _____ / 15

Problem 1, continued.

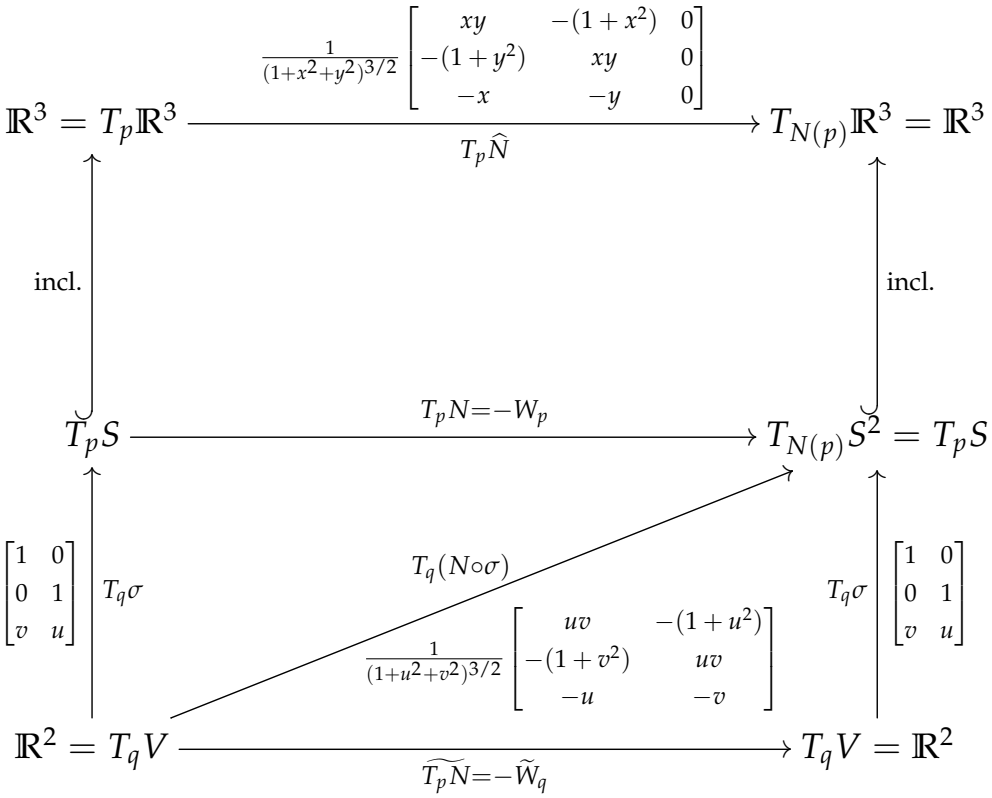
Define an ambient extension $\widehat{N}$ to $\mathbb{R}^3$ of the Gauss map N by

$$\widehat{N} : \mathbb{R}^3 \longrightarrow \mathbb{R}^3, \qquad \widehat{N}(x,y,z) = \frac{(-y,-x,1)}{\sqrt{1+x^2+y^2}}.$$

We then have the following commutative diagram:



Taking differentials in the previous diagram gives the commutative diagram



Problem 1, continued.

In the next few parts, you will compute the Weingarten map as a matrix in local coordinates.

1(b) **(25 points)** *Compute* $\widetilde{W}_q \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

[Hint: follow $(1,0)$ through $T_q\sigma$ and $T_p\widehat{N}$, or simply compute the image of $(1,0)$ under $T_q(N \circ \sigma)$. Express the result in the basis (σ_u, σ_v), i.e., the columns of the Jacobian matrix, and then apply the minus sign.]

Page subtotal: _____ / 25

Problem 1, continued.

1(c) **(20 points)** *Compute* $\widetilde{W}_q \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

1(d) **(5 points)** *Use the previous parts to compute* $\widetilde{W}_q$.

Page subtotal: _____ / 25

Problem 1, continued.

1(e) **(20 points)** *Find the Gaussian curvature $K(p)$ and mean curvature $H(p)$ in local coordinates.* [Hint: Use your coordinate Weingarten matrix $\widetilde{W}_q$]

Page subtotal: _____ / 20

Problem 1, continued.

1(f) **(15 points)** *What are the principal curvatures at $p = (1, 1, 1)$?*

$k_1 =$ _____, $k_2 =$ _____.

[Hint: Consider the vectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$]

Page subtotal: _____ / 15

Problem 1 total: _____ / 100