

MATH 4230/5230: Differential Geometry of Curves and Surfaces

Quiz 5

Friday, October 2, 2026

Name: _____

Score

____/10

Define $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ by

$$f(x_1, x_2) = (5x_1^2x_2^3, 2x_1 + x_2^2, x_1^2 - x_2^2).$$

Compute the Jacobian matrix of f at $p_0 = (1, -1)$. Show your work.

MATH 4230/5230 — Quiz 5 Solutions

The Jacobian matrix is

$$J(f) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} \end{pmatrix}.$$

The component functions are

$$\begin{aligned} f_1(x_1, x_2) &= 5x_1^2x_2^3, \\ f_2(x_1, x_2) &= 2x_1 + x_2^2, \\ f_3(x_1, x_2) &= x_1^2 - x_2^2. \end{aligned}$$

Their partial derivatives are

$$\begin{aligned} \frac{\partial f_1}{\partial x_1} &= 10x_1x_2^3, & \frac{\partial f_1}{\partial x_2} &= 15x_1^2x_2^2, \\ \frac{\partial f_2}{\partial x_1} &= 2, & \frac{\partial f_2}{\partial x_2} &= 2x_2, \\ \frac{\partial f_3}{\partial x_1} &= 2x_1, & \frac{\partial f_3}{\partial x_2} &= -2x_2. \end{aligned}$$

Thus,

$$J(f) = \begin{pmatrix} 10x_1x_2^3 & 15x_1^2x_2^2 \\ 2 & 2x_2 \\ 2x_1 & -2x_2 \end{pmatrix}.$$

At $(1, -1)$, the Jacobian matrix is

$$\boxed{\begin{pmatrix} -10 & 15 \\ 2 & -2 \\ 2 & 2 \end{pmatrix}}.$$