

MATH 4230/5230: Differential Geometry of Curves and Surfaces

Quiz 2

Friday, September 4, 2026

Name: _____

Score

____/10

Problem. Let $r > 0$, and consider the helix

$$\gamma(t) = (r \cos t, r \sin t, t).$$

Compute the Frenet frame $\mathbf{t}(t), \mathbf{n}(t), \mathbf{b}(t)$.

You may use

$$\mathbf{t} = \frac{\gamma'}{\|\gamma'\|}, \quad \mathbf{n} = \frac{\mathbf{t}'}{\|\mathbf{t}'\|}, \quad \mathbf{b} = \mathbf{t} \times \mathbf{n}.$$

MATH 4230/5230 — Quiz 2 Solutions

Since

$$\gamma'(t) = (-r \sin t, r \cos t, 1)$$

and

$$\|\gamma'(t)\| = \sqrt{r^2 \sin^2 t + r^2 \cos^2 t + 1} = \sqrt{r^2 + 1},$$

the unit tangent vector is

$$\mathbf{t}(t) = \frac{(-r \sin t, r \cos t, 1)}{\sqrt{r^2 + 1}}.$$

Next,

$$\mathbf{t}'(t) = \frac{(-r \cos t, -r \sin t, 0)}{\sqrt{r^2 + 1}}, \quad \|\mathbf{t}'(t)\| = \frac{r}{\sqrt{r^2 + 1}}.$$

Here we used $r > 0$. Therefore

$$\mathbf{n}(t) = (-\cos t, -\sin t, 0).$$

Finally,

$$\begin{aligned} \mathbf{b}(t) &= \mathbf{t}(t) \times \mathbf{n}(t) \\ &= \frac{1}{\sqrt{r^2 + 1}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -r \sin t & r \cos t & 1 \\ -\cos t & -\sin t & 0 \end{vmatrix} \\ &= \frac{(\sin t, -\cos t, r)}{\sqrt{r^2 + 1}}. \end{aligned}$$