

MATH 4230/5230

Differential Geometry of Curves and Surfaces

Quiz 1

Friday, August 28, 2026

Name: _____

Score

/ 10

Problem (10 points). Let

$$x = (0, 2), \quad y = (3, 4) \in \mathbb{R}^2.$$

Write x as a sum of two vectors, one parallel to y and the other orthogonal to y .

Show your work and clearly label your answers.

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Quiz 1 — Solutions

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For $x = (0, 2)$ and $y = (3, 4)$,

$$\langle x, y \rangle = 0 \cdot 3 + 2 \cdot 4 = 8, \quad \|y\|^2 = \langle y, y \rangle = 3^2 + 4^2 = 25.$$

The projection of x in the direction of y is

$$\text{proj}_y(x) = \frac{\langle x, y \rangle}{\langle y, y \rangle} y = \frac{8}{25} (3, 4) = \boxed{\left(\frac{24}{25}, \frac{32}{25} \right)}.$$

The projection onto the orthogonal complement is

$$x - \text{proj}_y(x) = x - \frac{\langle x, y \rangle}{\langle y, y \rangle} y = (0, 2) - \left(\frac{24}{25}, \frac{32}{25} \right) = \left(-\frac{24}{25}, \frac{18}{25} \right).$$

Therefore,

$$(0, 2) = \underbrace{\left(\frac{24}{25}, \frac{32}{25} \right)}_{\text{parallel to } y} + \underbrace{\left(-\frac{24}{25}, \frac{18}{25} \right)}_{\text{orthogonal to } y}.$$