Midterm 2

Linear Algebra: Matrix Methods

MATH 2130

Fall 2022

Friday October 28, 2022

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NAME: ____

PRACTICE EXAM

Question:	**1**	**2**	**3**	**4**	**5**	Total
Points:	20	20	20	20	20	100
Score:						

- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete this exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam as a single **.pdf** to **Canvas**, with the questions in the correct order, etc.
- You have 45 minutes to complete the exam.

1. • *Compute the determinant of each of the following matrices:*

(a) (10 points) $A = \begin{pmatrix} 4 & -1 & 1 \\ -1 & -2 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

(b) (10 points) $B = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & \pi \\ 1 & 0 & e & -4 & 8 & 3^{-5} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 5 & 1 & 0 & 2 & 10^4 \\ 0 & 0 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & -1 & 2 & 0 \end{pmatrix}$

1
20 points

2. • Let V be a real vector space and let $W_1, W_2 \subseteq V$ be two real subvector spaces of V. We define the *union* $W_1 \cup W_2$ of W_1 and W_2 to be the collection of vectors in V that are in W_1 or W_2:

$$W_1 \cup W_2 := \{\mathbf{v} \in V : \mathbf{v} \in W_1 \text{ or } \mathbf{v} \in W_2\}.$$

We define the *intersection* $W_1 \cap W_2$ of W_1 and W_2 to be the collection of vectors in V that are in W_1 and W_2:

$$W_1 \cap W_2 := \{\mathbf{v} \in V : \mathbf{v} \in W_1 \text{ and } \mathbf{v} \in W_2\}.$$

(a) (10 points) **True** or **False**: *If $W_1, W_2 \subseteq V$ are two real subvector spaces of a real vector space V, then the union $W_1 \cup W_2$ is a real subvector space of V.*

If true, provide a proof. If false, provide an example and prove that the example shows the statement is false. Your solution must start with the sentence "This statement is TRUE" or "This statement is FALSE".

(b) (10 points) **True** or **False**: *If $W_1, W_2 \subseteq V$ are two real subvector spaces of a real vector space V, then the intersection $W_1 \cap W_2$ is a real subvector space of V.*

If true, provide a proof. If false, provide an example and prove that the example shows the statement is false. Your solution must start with the sentence "This statement is TRUE" or "This statement is FALSE".

2
20 points

3. (20 points) • Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ be bases for a real vector space V, and suppose that

$$\begin{aligned}
\mathbf{v}_1 &= 4\mathbf{w}_1 - \mathbf{w}_2 + \mathbf{w}_3 \\
\mathbf{v}_2 &= 3\mathbf{w}_1 + 2\mathbf{w}_2 - \mathbf{w}_3 \\
\mathbf{v}_3 &= 7\mathbf{w}_1 + 23\mathbf{w}_2 - 2\mathbf{w}_3
\end{aligned}$$

Find the change-of-coordinates matrix to go from the coordinates with respect to the basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ *to the coordinates with respect to the basis* $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$.

3
20 points

4. (20 points) • Find a basis for the solution space of the difference equation

$$y_{k+2} + y_{k+1} - 56y_k = 0.$$

4
20 points

5. • Consider the following real matrix

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & -1 \\ 2 & 1 & 3 \end{pmatrix}$$

(a) (5 points) *Find the characteristic polynomial $p_A(t)$ of A.*

(b) (5 points) *Find the eigenvalues of A.*

(c) (5 points) *Find a basis for each eigenspace of A in $\mathbb{R}^3$.*

(d) (5 points) *Is A diagonalizable? If so, find a matrix $S \in \mathrm{M}_{3\times 3}(\mathbb{R})$ so that $S^{-1}AS$ is diagonal. If not, explain.*

5
20 points