

# Midterm 1

## Linear Algebra: Matrix Methods

MATH 2130

Fall 2025

Friday September 26, 2025

NAME: \_\_\_\_\_

## PRACTICE EXAM

Question:	1	2	3	4	5	Total
Points:	20	10	30	20	20	100
Score:						

- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete the exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam as a single **.pdf** to **Canvas**, with this **cover sheet**, and the questions in the correct order.
- You have 45 minutes to complete the exam.

1. (20 points) • Find all solutions to the following system of linear equations.

$$3x_1 + 9x_2 + 27x_3 = -3$$

$$-3x_1 - 11x_2 - 35x_3 = 5$$

$$2x_1 + 8x_2 + 26x_3 = -4$$

**Total for Question 1: 20**

2. (10 points) • Consider the linear map (“**transformation**”)  $L : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by

$$L(x_1, x_2, x_3) = (2x_1 - x_3, 3x_2 + x_3).$$

Write down the matrix form of (“**standard matrix for**”) the linear map  $L$ .

Total for Question 2: 10

3. • Consider the following matrix  $A$  and its corresponding Reduced Row Echelon Form matrix  $\text{RREF}(A)$ :

$$A = \begin{bmatrix} 1 & -3 & 0 & -1 & 4 & -2 \\ 0 & 0 & 1 & -1 & 0 & 1 \\ 3 & -9 & 0 & -3 & 2 & 4 \\ 1 & -3 & 1 & -2 & 4 & -1 \end{bmatrix} \quad \text{RREF}(A) = \begin{bmatrix} 1 & -3 & 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) (3 points) *Are the rows of  $A$  linearly independent?*

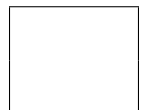
(b) (3 points) *Are the columns of  $A$  linearly independent?*

(c) (8 points) *Find a set of linearly independent vectors with the same span as the rows of  $A$ .*

(d) (8 points) *Find a set of linearly independent vectors with the same span as the columns of  $A$ .*

(e) (8 points) *Find a linearly independent set of vectors that span the set of solutions to the matrix equation  $A\mathbf{x} = \mathbf{0}$ .*

Total for Question 3: 30



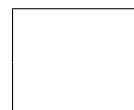
4. • Consider the matrix

$$B = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

- (a) (10 points) *Find the inverse of B.*

(b) (10 points) *Does there exist  $\mathbf{x} \in \mathbb{R}^3$  such that  $B\mathbf{x} = \begin{bmatrix} 5 \\ \sqrt{2} \\ \pi \end{bmatrix}$ ?*

Total for Question 4: 20



5. (20 points) • The equation

$$\mathbf{x} = C\mathbf{x} + \mathbf{d}$$

(the *Leontief Production Equation*) arises in the Leontief Input-Output Model. Here  $\mathbf{x}, \mathbf{d} \in M_{n \times 1}(\mathbb{R})$  are column vectors (called the *production vector* and the *final demand vector*) and  $C \in M_{n \times n}(\mathbb{R})$  is a square matrix (called the *consumption matrix*). Consider also the equation

$$\mathbf{p} = C^T \mathbf{p} + \mathbf{v}$$

(called the *Price Equation*), where  $\mathbf{p}, \mathbf{v} \in M_{n \times 1}(\mathbb{R})$  are column vectors (called the *price vector* and the *value added vector*).

Show that

$$\mathbf{p}^T \mathbf{d} = \mathbf{v}^T \mathbf{x}.$$

(This quantity is known as *GDP*) [Hint: Compute  $\mathbf{p}^T \mathbf{x}$  in two ways.]

Total for Question 5: 20

