

Exercise 5.2.24

Linear Algebra MATH 2130

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 5.2.24 from Lay [LLM21, §5.2]:

Exercise 5.2.24. Suppose that A and B are square matrices of the same size, and B is invertible.

True or False: *The matrices A and $B^{-1}AB$ have the same sets of eigenvalues.*

Solution. **True.** The eigenvalues of A and $B^{-1}AB$ are the real roots of the characteristic polynomials $p_A(t)$ and $p_{B^{-1}AB}(t)$. These two polynomials are the same:

$$\begin{aligned} p_{B^{-1}AB}(t) &= \det(tI - B^{-1}AB) = \det(B^{-1}B(tI) - B^{-1}AB) = \det(B^{-1}(tI)B - B^{-1}AB) \\ &= \det(B^{-1}(tI - A)B) = \det(B^{-1}) \det(tI - A) \det(B) = \det(B)^{-1} \det B \det(tI - A) \\ &= \det(tI - A) = p_A(t). \end{aligned}$$

□

REFERENCES

[LLM21] David Lay, Stephen Lay, and Judi McDonald, *Linear Algebra and its Applications*, Sixth edition, Pearson, 2021.

UNIVERSITY OF COLORADO, DEPARTMENT OF MATHEMATICS, CAMPUS BOX 395, BOULDER, CO 80309

Email address: `casa@math.colorado.edu`