**Exercise 4.5.9**

# Linear Algebra
# MATH 2130

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 4.5.9 from Lay [LLM21, §4.5]:

**Exercise 4.5.9.** Find the dimension of the subspace spanned by the vectors

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix}.$$

*Solution.* We can consider the matrix

$$\begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 2 & 1 & -2 & 1 \end{bmatrix}$$

with columns given by the given vectors. We put the matrix in RREF, and we obtain the matrix

$$\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Therefore, since there are two nonzero rows in the RREF, we see that the dimension of the subspace spanned by the given vectors is 2. □

*Remark* 0.1. Recall that a basis for the subspace spanned by the given vectors is given by the columns of the original matrix corresponding to the columns with the leading 1s in the RREF. In

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other words

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

form a basis for the subspace spanned by the given vectors.

*Remark* 0.2. One could also have done this problem by putting the vectors into the rows of a matrix. This would be the matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 1 \\ 9 & 4 & -2 \\ -7 & -3 & 1 \end{bmatrix}$$

One would then put this matrix in RREF:

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since there are two nonzero rows, one would also concluded that the dimension of the subspace spanned by the given vectors is 2. A basis for the subspace spanned by the given vectors is given by the nonzero rows of the RREF. In other words

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}$$

also form a basis for the subspace spanned by the given vectors.

## REFERENCES

[LLM21] David Lay, Stephen Lay, and Judi McDonald, *Linear Algebra and its Applications*, Sixth edition, Pearson, 2021.

UNIVERSITY OF COLORADO, DEPARTMENT OF MATHEMATICS, CAMPUS BOX 395, BOULDER, CO 80309

*Email address*: casa@math.colorado.edu