Exercise 2.9.4

Linear Algebra
MATH 2130

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 2.9.4 from Lay [LLM21, §2.9]:

Exercise 2.9.4. Let

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} -3 \\ 5 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} -7 \\ 5 \end{bmatrix}.$$

The vectors $\mathbf{b}_1, \mathbf{b}_2$ form a basis for $\mathbb{R}^2$. Find the coordinates of $\mathbf{x}$ with respect to this basis.

Solution. The coordinates of $\mathbf{x}$ with respect to the basis $\mathbf{b}_1, \mathbf{b}_2$ are $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$; in other words

$$\mathbf{x} = 5\mathbf{b}_1 + 4\mathbf{b}_2.$$

$$\begin{bmatrix} -7 \\ 5 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ -3 \end{bmatrix} + 4 \begin{bmatrix} -3 \\ 5 \end{bmatrix}.$$

Here is how to find the solution. We are trying to find real numbers α_1, α_2 so that

$$\mathbf{x} = \alpha_1\mathbf{b}_1 + \alpha_2\mathbf{b}_2.$$

If we let B be the matrix with columns $\mathbf{b}_1, \mathbf{b}_2$, i.e.,

$$B = \begin{bmatrix} 1 & -3 \\ -3 & 5 \end{bmatrix}$$

and we let $\mathbf{a} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$, then we are trying to solve the equation

$$B\mathbf{a} = \mathbf{x}$$

Date: October 5, 2025.

$$\begin{bmatrix} 1 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} -7 \\ 5 \end{bmatrix}.$$

We can solve this using row operations:

$$\left[\begin{array}{cc|c} 1 & -3 & -7 \\ -3 & 5 & 5 \end{array}\right] \mapsto \left[\begin{array}{cc|c} 1 & -3 & -7 \\ 0 & -4 & -16 \end{array}\right] \mapsto \left[\begin{array}{cc|c} 1 & -3 & -7 \\ 0 & 1 & 4 \end{array}\right] \mapsto \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 4 \end{array}\right]$$

□

References

[LLM21] David Lay, Stephen Lay, and Judi McDonald, *Linear Algebra and its Applications*, Sixth edition, Pearson, 2021.

University of Colorado, Department of Mathematics, Campus Box 395, Boulder, CO 80309

Email address: casa@math.colorado.edu