**Exercise 2.2.10**

**Linear Algebra**
**MATH 2130**

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 2.2.10 from Lay [LLM21, §2.2]:

**Exercise 2.2.10.** Use matrix algebra to show that if $A$ is invertible and $D$ satisfies $AD = I$, then $D = A^{-1}$.

*Solution.* Since $A$ is invertible, by definition, there exists a matrix $A^{-1}$ such that $A^{-1}A = AA^{-1} = I$. So if we are given $AD = I$, then we can multiply both sides of the equation by $A^{-1}$ to arrive at $A^{-1}AD = A^{-1}I$. We have

$$\begin{aligned} A^{-1} &= A^{-1}I && I \text{ is the multiplicative idenity} \\ &= A^{-1}AD && \text{from above} \\ &= ID && A^{-1} \text{ is the multiplicative inverse of } A \\ &= D && I \text{ is the multiplicative idenity} \end{aligned}$$

Therefore, $D = A^{-1}$, completing the proof. □

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## REFERENCES

[LLM21] David Lay, Stephen Lay, and Judi McDonald, *Linear Algebra and its Applications*, Sixth edition, Pearson, 2021.

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