Exercise 1.9.38

Linear Algebra
MATH 2130

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ABSTRACT. This is Exercise 1.9.38 from Lay [LLM21, §1.9]:

Exercise 1.9.38. Describe the possible echelon forms of the matrix form ("standard matrix") of a linear map ("transformation") $T : \mathbb{R}^4 \to \mathbb{R}^3$ that is surjective ("onto").

Solution. The possible echelon forms for such a matrix are:

$$\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \end{bmatrix}, \begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}, \begin{bmatrix} \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}, \begin{bmatrix} 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$$

where a $\blacksquare$ indicates a non-zero entry, and a $*$ indicates an arbitrary entry. Indeed, for T to be surjective ("onto"), the columns of the matrix form ("standard matrix") A of T must span $\mathbb{R}^3$; by [LLM21, Theorem 4 d., p.37], this means that A has a leading entry ("pivot") in every row. The matrices above are exactly the echelon form matrices with a leading entry ("pivot") in every row. □

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References

[LLM21] David Lay, Stephen Lay, and Judi McDonald, *Linear Algebra and its Applications*, Sixth edition, Pearson, 2021.

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