Exercise 1.1.20

Linear Algebra
MATH 2130

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 1.1.20 from Lay [LLM21, §1.1]:

Exercise 1.1.20. Determine if the system of equations is consistent. You do not need to completely solve the system of equations.

$$\begin{aligned} x_1 \quad &+ \quad 3x_3 \quad &= \quad 2 \\ x_2 \quad &- \quad 3x_4 \quad &= \quad 3 \\ -2x_2 \quad &+ \quad 3x_3 \quad + \quad 2x_4 \quad &= \quad 1 \\ 3x_1 \quad &+ \quad 7x_4 \quad &= \quad -5 \end{aligned}$$

Solution. The augmented matrix associated to the system of equations is

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{array}\right]$$

We can put the left hand side of the matrix in Row Echelon Form (REF) to determine if the system is consistent. To this aim, adding -3 times the first row to the last row we have

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 0 & 0 & -3 & 7 & -11 \end{array}\right]$$

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Then adding 2 times the second row to the third row we have

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & -3 & 7 & -11 \end{array}\right]$$

Finally, adding the third row to the fourth row, we have

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & 0 & 3 & -4 \end{array}\right]$$

The left hand side is in REF. Since there are no zero rows on the left hand side with non-zero entries on the right, the system is consisent. □

REFERENCES

[LLM21] David Lay, Stephen Lay, and Judi McDonald, *Linear Algebra and its Applications*, Sixth edition, Pearson, 2021.

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