Midterm I

Intro to Discrete Math

MATH 2001

Fall 2024

Friday September 27, 2024

NAME: _____

PRACTICE EXAM

Question:	**1**	**2**	**3**	**4**	**5**	Total
Points:	25	15	20	20	20	100
Score:						

- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete this exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam as a single **.pdf** to **Canvas**, with the questions in the correct order, etc.
- You have 45 minutes to complete the exam. **We will spend the last 5 minutes of class to upload your exam to Canvas.**

1. • Consider the sets $A = \{3, 9\}$ and $B = \{2, 3, 5\}$.

For this problem **you do <u>not</u> need to justify your answer**.

(a) (5 points) *List the elements of the power set* $\mathscr{P}(A)$.

(b) (5 points) *List the elements of the set* $A \times B$.

(c) (5 points) *List the elements of the set* $A \cup B$.

(d) (5 points) *Is it true that* $2 \in (A \cap B)$*?*

(e) (5 points) *List the elements of the set* $B - A$.

1
25 points

2. (15 points) • Suppose that A and B are finite sets. *What is* $|A \times B|$ *in terms of* $|A|$ *and* $|B|$*?* Explain.

2
15 points

3. (20 points) • For each $t \in \mathbb{R}$, consider the set

$$A_t = \{(x, y) \in \mathbb{R}^2 : t \leq x \leq t + 1 \text{ and } y = t\}.$$

Describe the union

$$\bigcup_{t \in [0,1]} A_t$$

as a geometric object in the plane $\mathbb{R}^2$. A good picture and a brief explanation of your solution is sufficient for this problem.

3
20 points

4. • Let $a \in \mathbb{R}$ and let f be a function from $\mathbb{R}$ to $\mathbb{R}$ (i.e., a function just like you are used to in all of your math classes, such as $f(x) = x^2$, $f(x) = \sin(x)$, etc.).

(a) (10 points) *Write the following sentence using only logical and mathematical notation* (e.g., the symbols $\wedge, \forall, \exists, \implies, \in, \mathbb{R}$, etc.). You may find it convenient to use the set $\mathbb{R}^+ := \{x \in \mathbb{R} : x > 0\}$.

For all real numbers M, there exists a positive real number δ such that for all real numbers x, if $0 < |x - a| < \delta$, then $f(x)$ is greater than M.

(b) (5 points) *Negate your answer to the previous part and simplify it as much as you can using standard logical equivalences.*

(c) (5 points) *Rewrite your answer to the previous part in plain english.*

4
20 points

5. • **True or False**. For this problem **you do <u>not</u> need to justify your answer**.

(a) (4 points) **True or False** (circle one). If A and B are finite sets and $A \cap B = \emptyset$, then $|A \cup B| = |A| + |B|$.

(b) (4 points) **True or False** (circle one). If A and B are subsets of a set X, then, regarding complements of subsets of X, we have $(A \cap B)^C = A^C \cup B^C$.

(c) (4 points) **True or False** (circle one). The negation of the statement $\forall x \in X,\ \exists y \in Y\ s.t.\ p(x,y)$ is logically equivalent to the statement $\exists x \in X\ s.t.\ \exists y \in Y\ s.t.\ \sim p(x,y)$.

(d) (4 points) **True or False** (circle one). The truth table for the statement $p \wedge q$ is

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	T
F	F	F

(e) (4 points) **True or False** (circle one). $(\mathbb{R} - \mathbb{Z}) \times \mathbb{N} = (\mathbb{R} \times \mathbb{N}) - (\mathbb{Z} \times \mathbb{N})$.

5
20 points