Final Exam

Intro to Discrete Math

MATH 2001

Fall 2024

Sunday December 15, 2024

NAME: ______________________________

PRACTICE EXAM

Question:	1	2	3	4	5	6	Total
Points:	20	20	20	20	20	20	120
Score:							

- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete this exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam as a single **.pdf** to **Canvas**, with the questions in the correct order, etc.
- You have 60 minutes to complete the exam.

1. (20 points) • The Fibonacci sequence $F_1,\ F_2,\ F_3, \ldots$ is defined by the rule that $F_1 = 1$, $F_2 = 1$, and for $n \geq 3$, one sets $F_n = F_{n-1} + F_{n-2}$. In other words, the Fibonacci sequence begins:

$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34, \ldots$$

Give a *proof by induction* that for each natural number n the following statement is true:

$$\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}.$$

Total for Question **1: 20**

2. (20 points) • **TRUE** or **FALSE**:

If R and S are equivalence relations on a set A, then $R \subseteq S$ if and only if for all $X \in A/R$ there exists $Y \in A/S$ with $X \subseteq Y$.

If true, give a *proof* of the statement. If false, provide a *counter example*, and prove that it is a counter example. Your solution must start with the sentence, "*This statement is TRUE*," or the sentence, "*This statement is FALSE*."

Recall that A/R is the set of equivalence classes for the equivalence relation R, and A/S is the set of equivalence classes for the equivalence relation S.

Total for Question **2: 20**

3. • Answer the following questions about maps of sets.

(a) (4 points) *Write down all the maps (functions) of sets $f : \{1,2\} \to \{1,2\}$ by listing the values of $f(1)$ and $f(2)$.*

1. $f(1) =$ $f(2) =$ 2. $f(1) =$ $f(2) =$ 3. $f(1) =$ $f(2) =$ 4. $f(1) =$ $f(2) =$

(b) (2 points) *Circle the maps above that are injective.*

(c) (2 points) *Are there any maps above that are injective but not surjective?*

(d) (5 points) *How many injective maps of sets $f : \{1,2,\ldots,n\} \to \{1,2,\ldots,n\}$ are there?* Explain.

(e) (2 points) *How many bijective maps of sets* $f : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ *are there?* Explain.

(f) (5 points) *If* A *and* B *are finite sets, how many bijective maps of sets* $f : A \to B$ *are there?* Explain.

Total for Question **3: 20**

4. • Consider the set $\Gamma = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + 4x - y = -4\}$, and the function $g : \mathbb{R} \to \mathbb{R}$ defined as $g(x) = x\sin(x)$.

(a) (5 points) *Show that Γ defines a map (function) $f : \mathbb{R} \to \mathbb{R}$?*

(b) (1 point) *What is $f(1)$?*

(c) (1 point) *Write a formula for $f(x)$.*

(d) (1 point) *What is the source (domain) of f?*

(e) (3 points) *What is the image of g?*

(f) (2 points) *Write formulas for* $g \circ f$ *and* $f \circ g$.

(g) (1 point) *Find* $(g \circ f)(1)$.

(h) (2 points) *Find* $(f \circ g)^{-1}(\{4\}) \cap \{x \in \mathbb{R} : g(x) \geq 0\}$.

(i) (2 points) *Is* $f \circ g$ *surjective?*

(j) (2 points) *Is* $g \circ f$ *surjective?*

Total for Question **4: 20**

5. (20 points) • Suppose that $\phi : A \to B$ and $\psi : B \to C$ are maps of sets. **TRUE** or **FALSE**:

If $\psi \circ \phi$ is surjective, then ψ is surjective.

If true, give a *proof* of the statement. If false, provide a *counter example*, and prove that it is a counter example. Your solution must start with the sentence, "*This statement is TRUE,*" or the sentence, "*This statement is FALSE*."

Total for Question **5: 20**

6. (20 points) • The function $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by the formula $f(x, y) = (3x + 5y, x + 2y)$ is bijective.

Find its inverse. You must show that your inverse is an inverse for f.

Total for Question **6: 20**